Detection templates for extreme mass ratio inspirals: Is the radiative approximation sufficient?

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Motivation

• LISA should observe $\sim 1000$ inspirals of compact ($\mu \sim 1 - 10 M_\odot$) objects into massive black holes ($M \sim 10^6 M_\odot$) (Gair et al. 2003). Last year of inspiral will contain $\sim M/\mu \sim 10^5$ cycles of waveform in the relativistic, near-horizon regime.

• Similar inspirals into intermediate mass black holes ($M \sim 10^3 M_\odot$) could be detected by advanced LIGO out to several hundred Mpc (Brown et al. 2006); event rate could be up to $\sim 10$/yr.
Motivation: Scientific Payoffs

• Precision test of general relativity in strong field regime. Measure multipole moments of central object (Ryan 1997, Li & Lovelace 2008), unambiguous identification as black hole.

• Measure hole’s mass and spin to $\sim 10^{-4}$ (Barack & Cutler 2004), constrain growth history (merger versus accretion) of black holes (Hughes & Blandford 2003).

• Learn about central parsec of galactic nuclei from event rate and distribution of inspiralling object’s masses.

• Potentially measure Hubble constant to 1 percent (MacLeod and Hogan 2008), indirectly aiding dark energy studies.

• However, all of these require templates with fractional phase accuracy of $\sim 10^{-5}$.
Approximation schemes for waveforms

- Geodesic equation in Kerr with self force can be written

\[
\frac{dq_\alpha}{dt} = \omega_\alpha(J) + \varepsilon \left[ \left\langle g^{(1)}_{\text{diss} \alpha} \right\rangle (J) + \delta g^{(1)}_{\text{diss} \alpha} (q, J) + g^{(1)}_{\text{cons} \alpha} (q, J) \right] + O(\varepsilon^2)
\]

\[
\frac{dJ_\lambda}{dt} = \varepsilon \left[ \left\langle G^{(1)}_{\text{diss} \lambda} \right\rangle (J) + \delta G^{(1)}_{\text{diss} \lambda} (q, J) + G^{(1)}_{\text{cons} \lambda} (q, J) \right] + \varepsilon^2 \left\langle G^{(2)}_{\text{diss} \lambda} \right\rangle (J) + \ldots
\]

where \( \varepsilon = \mu/M \), with solutions

\[
q_\alpha(t, \varepsilon) = \varepsilon^{-1} \psi^{(0)}_\alpha(\varepsilon t) + \varepsilon^{-1/2} \psi^{(1/2)}_\alpha(\varepsilon t) + \psi^{(1)}_\alpha(\varepsilon t) + \ldots
\]

\[
J_\lambda(t, \varepsilon) = \mathcal{J}^{(0)}_\lambda(\varepsilon t) + \varepsilon^{1/2} \mathcal{J}^{(1/2)}_\lambda(\varepsilon t) + \varepsilon \mathcal{J}^{(1)}_\lambda(\varepsilon t) + \varepsilon \mathcal{H}_\lambda(\mathcal{J}^{(0)}_\lambda, q_\alpha) \ldots
\]
Approximation schemes for waveforms

- Geodesic equation in Kerr with self force can be written

\[
\frac{dq_\alpha}{dt} = \omega_\alpha(J) + \varepsilon \left[ \left\langle g^{(1)}_{\text{diss}\,\alpha} \right\rangle (J) + \delta g^{(1)}_{\text{diss}\,\alpha}(q, J) + g^{(1)}_{\text{cons}\,\alpha}(q, J) \right] + O(\varepsilon^2)
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where \( \varepsilon = \mu/M \), with solutions

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\]

\[
J_\lambda(t, \varepsilon) = \mathcal{J}^{(0)}(\varepsilon t) + \varepsilon^{1/2} \mathcal{J}^{(1/2)}(\varepsilon t) + \varepsilon \mathcal{J}^{(1)}(\varepsilon t) + \varepsilon \mathcal{H}_\lambda(\mathcal{J}^{(0)}(t), q_\alpha) \ldots
\]

Adiabatic Approximation:

\[
\frac{d\psi^{(0)}_\alpha}{d\tilde{t}} = \omega_\alpha(\mathcal{J}^{(0)}), \quad \frac{d\mathcal{J}^{(0)}_\lambda}{d\tilde{t}} = \left\langle G^{(1)}_{\text{diss}\,\lambda} \right\rangle (\mathcal{J}^{(0)}).
\]

- Dissipative piece of 1st order self force known in principle (Mino 2003, Sago et. al 2006, Sundararajan et al., in prep.), whereas conservative piece not yet known
- Action variables evolve independently
- Not equivalent to using self-force computed from \( \left\langle dJ_\lambda/dt \right\rangle \)
Approximation schemes for waveforms

- Geodesic equation in Kerr with self force can be written

\[
\frac{dq_\alpha}{dt} = \omega_\alpha(J) + \varepsilon \left[ \left\langle g_{\text{diss} \alpha}^{(1)} \right\rangle (J) + \delta g_{\text{diss} \alpha}^{(1)} (q, J) + g_{\text{cons} \alpha}^{(1)} (q, J) \right] + O(\varepsilon^2)
\]

\[
\frac{dJ_\lambda}{dt} = \varepsilon \left[ \left\langle G_{\text{diss} \lambda}^{(1)} \right\rangle (J) + \delta G_{\text{diss} \lambda}^{(1)} (q, J) + G_{\text{cons} \lambda}^{(1)} (q, J) \right] + \varepsilon^2 \left\langle G_{\text{diss} \lambda}^{(2)} \right\rangle (J) + \ldots
\]

where \( \varepsilon = \mu / M \), with solutions

\[
q_\alpha(t, \varepsilon) = \varepsilon^{-1} \psi_\alpha^{(0)}(\varepsilon t) + \varepsilon^{-1/2} \psi_\alpha^{(1/2)}(\varepsilon t) + \psi_\alpha^{(1)}(\varepsilon t) + \ldots
\]

\[
J_\lambda(t, \varepsilon) = \mathcal{J}_\lambda^{(0)}(\varepsilon t) + \varepsilon^{1/2} \mathcal{J}_\lambda^{(1/2)}(\varepsilon t) + \varepsilon \mathcal{J}_\lambda^{(1)}(\varepsilon t) + \varepsilon \mathcal{H}_\lambda(\mathcal{J}_\lambda^{(0)}, q_\alpha) \ldots
\]

Post-1/2-Adiabatic:

- Due to resonances
- Requires knowledge of all pink forcing terms
- Does not arise for circular or equatorial orbits
- See talk by Tanja Hinderer in session L11 this afternoon.
Approximation schemes for waveforms

- Geodesic equation in Kerr with self force can be written

\[
\frac{dq_\alpha}{dt} = \omega_\alpha(J) + \varepsilon \left[ \left\langle g^{(1)}_{\text{diss } \alpha} \right\rangle(J) + \delta g^{(1)}_{\text{diss } \alpha}(q, J) + g^{(1)}_{\text{cons } \alpha}(q, J) \right] + O(\varepsilon^2)
\]

\[
\frac{dJ_\lambda}{dt} = \varepsilon \left[ \left\langle G^{(1)}_{\text{diss } \lambda} \right\rangle(J) + \delta G^{(1)}_{\text{diss } \lambda}(q, J) + G^{(1)}_{\text{cons } \lambda}(q, J) \right] + \varepsilon^2 \left\langle G^{(2)}_{\text{diss } \lambda} \right\rangle(J) + \ldots
\]

where \( \varepsilon = \mu/M \), with solutions

\[
q_\alpha(t, \varepsilon) = \varepsilon^{-1} \psi^{(0)}_\alpha(\varepsilon t) + \varepsilon^{-1/2} \psi^{(1/2)}_\alpha(\varepsilon t) + \psi^{(1)}_\alpha(\varepsilon t) + \ldots
\]

\[
J_\lambda(t, \varepsilon) = \mathcal{J}^{(0)}_\lambda(\varepsilon t) + \varepsilon^{1/2} \mathcal{J}^{(1/2)}_\lambda(\varepsilon t) + \varepsilon \mathcal{J}^{(1)}_\lambda(\varepsilon t) + \varepsilon \mathcal{H}_\lambda(\mathcal{J}^{(0)}_\lambda, q_\alpha) + \ldots
\]

Post-1-Adiabatic:

\[
\frac{d\psi^{(1)}_\alpha}{d\tilde{t}} = \left\langle g^{(1)}_\alpha \right\rangle + \frac{\partial \omega_\alpha}{\partial J_\lambda} \mathcal{J}^{(1)}_\lambda, \quad \frac{d\mathcal{J}^{(1)}_\lambda}{d\tilde{t}} = \left\langle G^{(2)}_{\text{diss } \lambda} \right\rangle + "\text{beating terms}".
\]

- Requires knowledge of all pink and blue forcing terms
- Gives phase correct to \( O(1) \).
Approximation schemes for waveforms

- Geodesic equation in Kerr with self force can be written

\[ \frac{dq_\alpha}{dt} = \omega_\alpha(J) + \varepsilon \left[ \left\langle g^{(1)}_{\text{diss } \alpha} \right\rangle(J) + \delta g^{(1)}_{\text{diss } \alpha}(q, J) + g^{(1)}_{\text{cons } \alpha}(q, J) \right] + O(\varepsilon^2) \]

\[ \frac{dJ_\lambda}{dt} = \varepsilon \left[ \left\langle G^{(1)}_{\text{diss } \lambda} \right\rangle(J) + \delta G^{(1)}_{\text{diss } \lambda}(q, J) + G^{(1)}_{\text{cons } \lambda}(q, J) \right] + \varepsilon^2 \left\langle G^{(2)}_{\text{diss } \lambda} \right\rangle(J) + \ldots \]

where \( \varepsilon = \mu/M \), with solutions

\[ q_\alpha(t, \varepsilon) = \varepsilon^{-1} \psi^{(0)}_\alpha(\varepsilon t) + \varepsilon^{-1/2} \psi^{(1/2)}_\alpha(\varepsilon t) + \psi^{(1)}_\alpha(\varepsilon t) + \ldots \]

\[ J_\lambda(t, \varepsilon) = J^{(0)}_\lambda(\varepsilon t) + \varepsilon^{1/2} J^{(1/2)}_\lambda(\varepsilon t) + \varepsilon J^{(1)}_\lambda(\varepsilon t) + \varepsilon \mathcal{H}_\lambda(J^{(0)}_\lambda, q_\alpha) + \ldots \]

Radiative Approximation:

- Use only radiative (dissipative) piece of self force (Mino 2003)
- Gives adiabatic waveforms plus a piece of post-1-adiabatic corrections
- Will the errors impede signal detection with LIGO/LISA?
Studies of Radiative Approximation

• All studies use post-Newtonian approximation to conservative pieces of self-force to get a rough estimate of phase error.

• Early studies for LISA: PN equations of motion, circular, equatorial orbits (Burko 2003, Drasco et al. 2005). Extended to finite eccentricity (Favata 2006). Phase errors typically <1 cycle.

• Similar study for LIGO (Brown et al. 2006) estimated 10 percent reduction in signal-to-noise ratio.


• Parameter estimation errors studied by Huerta and Gair (2008) and by Drasco et al. (2009), next talk.
Results of Pound and Poisson

\[ p \rightarrow 0.9p, \ e_0 = 0.9, \ \mu/M = 0.1 \]

\[ p_0 = 50, \ e_0 = 0.9 \]

- Shows effect is large in weak field regime. But what about last year of inspiral for LISA sources?
Our Results

- Max orbital phase error in last year of inspiral. True and approximate waveforms are lined up at some time $t$ which is optimized over. Initial data chosen so secular pieces coincide.

- Conclusion: likely good enough for detection templates, but further study required.