Special Relativity

Lecture 17

Relativity

• There is no absolute motion.
• Everything is relative.
• Suppose two people are alone in space and traveling towards one another
  • As measured by the Doppler shift!
    – Which one is moving?
    – They can’t tell!
Example:

• A train is moving at 65 mph relative to the tracks.
• If the people inside the train cannot see out and the track is very smooth, they can not tell they are moving!

• The earth moves around the sun at 30 km/sec. Can you tell?

Special Theory of Relativity

• Postulates:
  – Albert Einstein (1905)
  1. The speed of light is the same to all observers, irrespective of their motion.

  2. The laws of physics are the same everywhere no matter what the speed of the observer.
Things don’t add up the way they used to.

• For instance, a boater throws a ball with velocity $v_{\text{ball}}$ (as seen by the boater):

![Diagram showing a ball being thrown by a boat and viewed from the shore]

Standing on the shore we see the ball moving with velocity $= v_{\text{boat}} + v_{\text{ball}}$

Addition of Velocities
The Lorentz Transformation

$$V(1,2) = \frac{V_1 + V_2}{1 + \frac{V_1 V_2}{c^2}}$$

When $V_1 = V_2 = c$, then:

$$V(1,2) = \frac{c + c}{1 + \frac{c^2}{c^2}}$$

$$= \frac{2c}{2}$$

$$= c$$
The speed of light is constant.

- But this doesn’t happen with a beam of light!

- The velocity of light as seen from on shore is still $c$.

---

**Results of the Theory**

Addition of Velocities

\[
\begin{align*}
\text{Car 1:} & \quad 200 \text{ km/hr} \\
\text{YOU:} & \quad \text{at rest} \\
\text{Car 2:} & \quad 200 \text{ km/hr}
\end{align*}
\]

\[
\begin{align*}
\text{Speed(1, YOU)} & = 200 \text{ km/hr} \\
\text{Speed(2, YOU)} & = 200 \text{ km/hr} \\
\text{Speed(1, 2)} & = 400 \text{ km/hr}
\end{align*}
\]
Results of the Theory
Addition of Velocities

SIMULTANEITY

Two events simultaneous in one reference frame are not simultaneous in another frame moving relative to the first.
## Special Theory of Relativity

<table>
<thead>
<tr>
<th>Fraction of Speed of Light (Relative Velocity)</th>
<th>Length Contraction Factor</th>
<th>Mass Increase Factor</th>
<th>Time Dilation Factor</th>
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### “TACHYONS”

There once was a lady called Bright,  
Who could travel faster than light.  
She went out one day,  
*In a relative way,*  
And came back the previous night!
Predictions of Special Relativity

Measurements of **TIME**, **LENGTH**, **MASS** depend on the velocity of the observer.

With increasing speed:

- **Lengths shrink**
- **Masses increase**
- **Clocks slow down**

How do we add velocities?

- The old law of Galileo and Newton was
  \[ V_{\text{TOTAL}} = V_1 + V_2 \]

  - If \( V_1 = c \) (flashlight) and \( V_2 = V_{\text{boat}} \) then
    \[ V_{\text{TOTAL}} = c + V_{\text{boat}} > c \]

  - **Wrong!!** Can’t have \( V > c \). We need a new way of adding velocities.
The Lorentz Transformation

• A new law for the addition of velocities

\[ V_{\text{TOTAL}} = \frac{V_1 + V_2}{1 + \frac{V_1 V_2}{c^2}} \]

- If \( V_1 = c \) (flashlight) and \( V_2 = V_{\text{boat}} \) then

\[ V_{\text{TOTAL}} = \frac{c + V_2}{1 + \frac{c V_2}{c^2}} = \frac{c + V_2}{1 + \frac{V_2}{c}} = \frac{c}{c + \frac{V_2}{c}} = c \]

Another example

- Person C sees A and B moving at 0.9c.
- How fast does A think B is moving?

\[ V_{\text{TOTAL}} = \frac{0.9c + 0.9c}{1 + \frac{0.9c \times 0.9c}{c^2}} = \frac{1.8c}{1 + 0.81} = 0.9945c \]
Simultaneity

- The simultaneity of events is in the eye of the beholder.

- A light bulb in the center of a high speed train flashes. An observer on the train sees the light reach the front and back simultaneously.

From outside the train

- To the outside observer the light reaches the back first.
- The back moves towards the light and the front away.
- Not simultaneous!

Flash goes off
Flash reaches back wall first
Predictions of special relativity

• Measurement of
  
  TIME  LENGTH  MASS
  
  depend on the velocity of the observer.

• As velocity increases
  
  - Lengths shrink (contract)
  - Masses increase
  - Clocks slow down

Time Dilation

Consider a train moving with a velocity, \( v \)

- Inside the train is a “clock” which consists of two mirrors separated by a distance \( d \) with a light beam bouncing back and forth.

  - Every time the photon hits a mirror, we get a “tick” of the clock.
To a person sitting on the train, the time between ticks is:

\[ t = \frac{d}{c} \]

What does a person outside the train see?

To the person outside the train, the distance between the mirrors will have changed!

If \( t' \) is the time between ticks as seen by the person outside, then the mirror will have moved a distance \( vt' \).
Then
\[ d'^2 = d^2 + v^2 t'^2 \]
or
\[ c^2 t'^2 = c^2 t^2 + v^2 t'^2 \]
solving for \( t' \)
\[ t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} \]

... Time Dilation

- So we have:
\[ t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} \]

- Time measured by a person on the train
- \( t' \) = time measured by a person beside the train

- Since \((1 - \frac{v^2}{c^2}) < 1\), the time between ticks is greater for the person beside the train.

- The “clock” on the train appears to run slower to the person outside!!!
  - In fact, it does run slower – decaying particles take longer to decay when they move at a high speed compared to the lab.
The Tricky Part? ... Time Dilation

- On the train, a “Timex” must keep the same time as the photon clock.
- Relativity says that the person on the train can’t tell she is moving.
  - ⇒ the Timex must slow down too.
    - Same for a Rolex, a CD player, her heart beat (!), etc.
    - Otherwise physics would be different (you could pull all the shades on the train and still tell you were moving – but relative to what!)
- **Everything slows down!!!**

**Curve shows “aging” of moving object**
- That is, time passes more slowly
- At v = c, the clock appears to stop !!!

![Time Dilation Graph](image.png)
**Length Contraction**  (Can be derived similarly)

Consider now two photon clocks (1 and 2 below) oriented perpendicular to one another on the train.

- To an observer on the train $d = d'$.  
- What about an outside observer?

\[
L = L_o \sqrt{1 - \frac{v^2}{c^2}}
\]

- Lengths shrink in the direction of motion
  - $L_o = $ rest length,  $L = $ observed length
- At $v = c$, $L = 0$ !!!
Mass increases with velocity.
- $m_o$ = rest mass, $m$ = observed mass
- At $v = c$, the mass is infinite !!!

Energy and Mass

- Einstein’s formula $E = mc^2$
- can now be written as
  $$E = \frac{m_o c^2}{\sqrt{1 - v^2 / c^2}}$$
- Problem for interstellar travel
  - Getting hit by a dust particle would be dangerous if traveling at high speeds.
The “Twin Paradox”

• One twin travels away from the Earth at high speed for a long time.
• Returning to Earth she finds herself much younger than her brother!
• How?
• Both twins think each other’s clock slows down, so what is going on?

No “warp” drive

The view from the rocket

• Suppose one twin is on a rocket ship traveling at 0.99c to a star 25 lyr away.
• She sees the distance contracted to:
  \[ L = 25 \times \sqrt{1 - (0.9c)^2} \text{ / } c^2 \text{ lyr} = 3.53 \text{ lyr} \]
  
  She computes her time to get there and back as \( 2 \times (3.53 \text{ lyr} / 0.99) = 7.13 \text{ years} \)
The view from Earth

• The twin on Earth sees the distance as still 25 lyr, so he computes her time to go out and back as:

\[
2 \times 25 \text{ lyr} / 0.99 = 50.51 \text{ years}
\]

• So he ages 50.51 years, while his twin sister ages only 7.13 years!!!

Faster than light?

• No normal matter can travel faster than the speed of light!

\[
m = \frac{m_o}{\sqrt{1 - v^2 / c^2}}
\]

- Tachyons -
  - **Theoretical** particles that **always** travel faster than the speed of light
  - Allowed in Special Relativity
  - Time travel paradoxes
What you should know about SR

• Postulates
  – The speed of light is the same to all observers, irrespective of their motion.
  – The laws of physics are the same everywhere no matter what the speed of the observer.

• Some consequences
  – Velocities add so that v < c.
  – Simultaneity of events depends on observer

• Physical Results
  – Time dilation, length contraction, mass increase with increasing velocity