1. Jupiter has alternating jets in its atmosphere that can be detected in the visible clouds near the one bar pressure level. Just above this level, where the pressure is about 0.5 bar and the temperature is about 140 K, infrared remote sensing reveals that latitudinal temperature gradients are systematically associated with the jets. In the northern hemisphere, eastward jets have positive latitudinal gradients capping them. A typical jet magnitude is 50 m s\(^{-1}\) and a typical temperature gradient above an eastward jet is 3 K per 2,000 km. Verify that the jets decay upward at the height sampled by the infrared instrument. Make an estimate of how high the jets extend, in scale heights, before they vanish.

2. Inside a semi-infinite slab of material (atmosphere, perhaps) the radiative source function \(J\) increases with depth according to
\[
J = J_0 + J_1 \tau,
\]
where \(J_0\) and \(J_1\) are constants and \(\tau\) is the normal, or perpendicular, optical depth, zero at the edge and increasing inward. An example of this behavior of \(J\) would be a gray atmosphere in radiative equilibrium. But do NOT investigate that particular case, in the present problem \(J_0\) and \(J_1\) are just arbitrary constants.

(a) Evaluate the emergent intensity \(I(\tau = 0, \mu)\) in terms of \(J_0\) and \(J_1\).

(b) Evaluate the emergent flux in terms of \(J_0\), \(J_1\) and \(\pi\). Yes, including \(\pi\) here is a hint!

(c) The emergent flux is equivalent to that from an optically thick surface characterized by a uniform \(J_S\). At what optical depth is \(J\) inside the semi-infinite slab equal to the “equivalent” surface radiance?

4. Under a variation of the two-stream approximation, two intensities are introduced, an upward \(I^\uparrow\) and a downward \(I^\downarrow\). \(I^\uparrow\) is assumed constant in the upper hemisphere of “direction” space and \(I^\downarrow\) in the lower hemisphere. (a) Verify that the net flux is then
\[
F = \pi \left( I^\uparrow - I^\downarrow \right). \tag{1}
\]

(b) Write two equations of transfer, for one for the upward beam and one for the downward beam. Consider only thermal emission and write \(B\) as the source function. Neglect scattering. Make your transfer equations as simple as possible. I would suggest, for example, setting cosine of the zenith angle equal to one. Some texts do this case for simplicity.

(c) Eliminate the intensities from (1) and your two transfer equations, and find the differential equation governing \(F(\tau)\). Collect \(F\)’s on the left side and \(B\)’s on the right hand side to put it in familiar form.
Next, apply the equation to an idealized atmosphere in radiative equilibrium with a constant flux passing through it. Neglect sunlight. A case in point might be one of the outer planets with an internal heat source. Assume $I^\downarrow = 0$ at the top, where $\tau = 0$. Let the absorption coefficient be independent of wavelength, so that there is a unique $\tau$ (and $\pi B = \sigma T^4$). Deep in the atmosphere simply let $\tau \to \infty$ and do not consider a surface.

(d) Find $B$ in terms of $F$ and $\tau$. 
Information

The complete set of pressure coordinate $\beta$-plane governing equations is given below.

\[
\begin{align*}
\frac{Du}{Dt} - fv + \frac{\partial \Phi}{\partial x} &= F_x, \\
\frac{Dv}{Dt} + fu + \frac{\partial \Phi}{\partial y} &= F_y, \\
-p \frac{\partial \Phi}{\partial p} &= RT, \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} &= 0, \\
\frac{DT}{Dt} - \frac{RT}{c_p} \frac{\partial \omega}{\partial p} &= \frac{Q}{c_p},
\end{align*}
\]

where $f = f_0 + \beta y$, $D/Dt = \partial/\partial t + u \partial/\partial x + v \partial/\partial y + \omega \partial/\partial p$, $\Phi = gz$, and other notation is standard.

$\Omega_J = 1.75 \times 10^{-4}$ s$^{-1}$

$\pi B = \sigma T^4$

$R = 8.3145$ J K$^{-1}$ mole$^{-1}$

Jupiter $[\text{H}_2]/([\text{H}_2] + [\text{He}]) = 0.86$

Midlatitude $g_J = 25$ m s$^{-2}$.

\[
\mu \frac{\partial I}{\partial r} = I - J \quad F_\nu = \int_0^{2\pi} \int_{-1}^1 I_\nu \cos \theta \ d(\cos \theta) \ d\phi
\]