A575/EAS575 Problem Set **Due 3 October 2006**

1. In lecture we wrote the formal integral of the equation of transfer for non-scattering infrared radiation as

$$I_\nu(\tau_\nu = 0) = I_\nu(\tau_{\nu S}) \exp(-\tau'_\nu) + \int_0^{\tau_{\nu S}} B_\nu(\tau'_\nu) \exp(-\tau'_\nu) d\tau'_\nu. \tag{1}$$

Here I is the upward radiance or intensity (vertical), and τ is the optical depth, increasing downward from zero at the top of the atmosphere. With a subscript S it denotes optical depth all the way sown to the planetary surface. From here on in this problem, drop the ν subscripts but let it be understood that the appropriate quantities are functions of frequency.

The transmissivity is given by

$$\mathcal{T} = \exp(-\tau).$$

Show that the emission to space can be written

$$I = I_S \mathcal{T}_S + \int_{\mathcal{T}_S}^{1} B \, d\mathcal{T}. \tag{2}$$

Here $\mathcal{T}$ and B have been regarded as functions of τ, the optical depth. But someone building a retrieval or inverse algorithm to deduce the temperature profile would usually prefer to see projection kernels that are functions of height or atmospheric pressure level, and not functions of frequency.

The shape of the kernel or weighting function will depend on the vertical coordinate used. This is because the weighting function measures the contribution per unit length along this coordinate, which clearly must depend on how much the coordinate is stretched. Show that in terms of a vertical coordinate h, arbitrary except that it increases upward, the formal solution (2) gives

$$\begin{aligned} I &= B(\text{surface}) \, \mathcal{T}(\text{surface}) + \int_0^\infty B \, \frac{d\mathcal{T}}{dh} \, dh \\ &= B_S \mathcal{T}_S + \int_0^\infty B \, K \, dh. \end{aligned}$$

where K is the kernel or weighting function that determines the effect on outgoing radiance of emission at different heights in the atmosphere. Here it has been assumed that the Planck function gives the intensity of emission from the surface.

Now suppose that the optical depth is due to absorption by some atmospheric gas that is well mixed and has an absorption cross-section independent of pressure p. Further, assume an isothermal atmosphere. Show that

$$I = B_S \mathcal{T}_S + \int_0^\infty B \exp\left(-\tau_0 \frac{p}{p_0}\right) \frac{p}{p_0} \tau_0 \frac{dz}{H},$$

where H is the scale height. A subscript 0 denotes evaluation at the planetary surface. Sketch or plot a few kernel functions. What does it take to place the kernel maximum high in the atmosphere?

2. At the visible level in the Venus clouds the droplet density is about 50 cm^{-3} and their shape is spherical. The mean radius is 1 μm. How far will visibility permit good imaging? Assume an optical cross-section equal to the geometric cross-section. For simplicity, think only about the horizontal viewing case.

3. What is the column abundance of the atmospheres of Venus, Earth, Mars and Titan, expressed in "precipitable meters", as defined by Dr. Toigo during his guest lecture?

4. The Clausius-Clapeyron equation gives

$$\frac{dp_{vs}}{dT} = \frac{\rho_v L_v}{T},$$

where the notation is standard, subscript v refers to the condensing vapor, and subscript s refers to saturation. For water on Earth and Mars, assume a constant latent heat $L = 3 \times 10^6$ J kg^{-1}. Show that with this assumption,

$$p_{vs} = p_0 \exp\left[\frac{L}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right], \tag{3}$$

where p_0 is the saturation vapor pressure at T_0. Then assume the temperature in the lower atmosphere of Mars is

$$T = T_0 - \left|\frac{dT}{dz}\right| z,$$

where dT/dz is approximately constant, say -2.5 K km^{-1}. Show that the saturation vapor pressure profile is approximately

$$p_v = p_{v0} \exp(-z/H_{vs}), \qquad \text{where} \qquad \frac{1}{H_{vs}} = \frac{L}{R_v T_0^2}\left|\frac{dT}{dz}\right|.$$

Evaluate H_{vs} and compare it with the usual pressure scale height.

Next, estimate the column abundance of water by assuming that the density profile is similar to the vapor profile in height. This is not a bad assumption for the saturated gas, because the saturation vapor pressure is a much stronger function than the temperature. (This mysterious remark should become clear when you begin writing things out.) You will need a numerical value for p_{v0}, which you can estimate from (3) and the fact that the triple point of water is 6.1 mb, 273.16 K. Observed column abundances of water vapor are 10 - 40 precipitable microns.

5. Take a look at water on Earth for comparison, In particular, estimate the scale height and the column saturated vapor amount. This time you will find it more convenient to use precipitable centimeters as the unit. Your answer will give you the maximum precipitation that can occur without convergence or advection supplying new vapr.

Use $dT/dz =$ -6 K km^{-1}, Make your own estimates of T_0 and whatever else you need.

6. The latent heat of CO_2 is about 5.7×10^5 J kg^{-1}. The surface temperature in the polar regions during winter is about 145 K. Assume an emissivity of unity and negligible back-radiation from the atmosphere, and estimate the depth of CO_2 snow that accumulates in a winter. Assume a snow/ice density of 0.3 gm cm^{-3}