1. A certain cloud covered planet appears homogeneous in its cloud properties, with the single scattering albedo, \( \varpi \), and the total cloud optical depth, \( \tau_c \), both well mixed and uniform. The surface of the planet is black, a perfect absorber at all wavelengths.
Assume isotropic scattering and use the two stream approximation. Find the contours in the \( \varpi_0, \tau_c \) plane along which the albedo is constant. An analytic expression is sufficient but a plot would be even better.
What observation would remove the ambiguity and determine both \( \varpi_0 \) and \( \tau_c \)?

2. A homogeneous cloud 500 m thick has infrared optical depth 5. The cloud is approximately isothermal with temperature 275 K. The surface, a couple of km below, is at temperature 290 K. Assume that the surface emits as a black body, the cloud is a gray absorber (no scattering), and the atmospheric gases can be treated as transparent. Since the cloud is gray, \( \pi B = \sigma T^4 \).
(a) Find an expression for the heating rate, \(-\partial F/\partial z\), in units W m\(^{-3}\), through the cloud. (b) Use
\[
\rho c_p \frac{\partial T}{\partial t} = -\frac{\partial F}{\partial z},
\]
to evaluate the heating rate in degrees K per hour. Assume a density \( \rho = 1 \) kg m\(^{-3}\). Plot the profile of \( dT/dt \) through the depth of the cloud. What do you think will happen within this cloud?
Hint: If you like, you can set up a grid and numerically invert the tridiagonal matrix arising from the difference form of the differential equation. But the analytic solution is easy and much less trouble.

3. Consider a semi-infinite atmosphere which absorbs both in the solar part of the spectrum and also in the infrared. The absorption is gray in each spectral region, but with different absorption coefficients. The solar radiation optical depth \( \tau_S \) is given in terms of the longwave optical depth \( \tau \) by \( \tau_S = \alpha \tau \). The global average solar flux is given by
\[
F_S = -F_0 \exp(-\tau_S/\mu_0),
\]
where \( F_0 \) is the planetary average absorbed flux, which takes into account albedo, distance from the Sun, and spherical geometry. Assume \( \mu_0 = 1/\sqrt{3} \).
The atmosphere is optically homogeneous and semi-infinite. The limit \( \tau \gg 1 \) can be considered without encountering any surface interface.
Assume radiative equilibrium and solve in the two-stream approximation for the profile \( B(\tau) \). Consider the limits \( \alpha \ll 1 \) and \( \alpha \gg 1 \). Which is the “greenhouse” case and which is the “thermosphere” case? Sketch \( B(\tau) \) for an example of each case. What happens when the solar and planetary absorption coefficients are equal?
4. Consider a 2D cosine jet in an east-west channel bounded on the north and south by walls at ±L. The flow initially has the form

\[ u_A = u_0 + \left( 1 + \cos \frac{\pi y}{L} \right) u_J, \]

where \( u_0 \) gives the overall offset of the profile, and \( u_J \) gives the strength of the shear. Suppose some mixing event occurs, leading to a final state \( u = u_B \), where \( u_B \) is a constant. Momentum is conserved during the mixing event. What is the kinetic energy difference between state \( B \) and state \( A \)? Express the answer per unit depth and per unit area, since you are not given depth and length.

5. Suppose that the jet stability is determined by the requirement that \( \beta - \partial^2 u / \partial y^2 \) change sign somewhere along the profile. Consider the two cases \( u_J > 0 \) and \( u_J < 0 \). Let \( u_J \) increase in magnitude from zero, at which point the jet is obviously stable. Where along the jet profile will the sign change first occur in the two cases?