1. Consider a radio source for which a brightness temperature \( T_b \) has been estimated. In class it was stated but not proven that, for incoherent sources, \( T_b \) is related to the mean particle energy in the radiating source, subject to several caveats. Adopting this relation, estimate the ratio of mean electron energy and rest-mass energy \( m_e c^2 \) for sources with brightness temperatures of \( 10^4 \) K, \( 10^{12} \) K and \( 10^{30} \) K. Are the energy estimates you get in the three cases realistic? In the third case, compare the energy you get with the energy of the most energetic cosmic rays that have been measured.

2. Consider an active galactic nucleus whose angular size is 1 mas and whose flux density is 1 Jy at 1 GHz. Estimate its brightness temperature. Also estimate the size of aperture needed to resolve the source.

3. Estimate the brightness temperature of a ‘nano-pulse’ from the Crab pulsar by referring to the attached plots, which show a single pulse from the Crab pulsar that contains a train of individual nano-pulses. The lower panels labeled a-f are zoom-ins of particular nano-pulses in the upper panel. The width of each lower panel is 0.1 microsec. Note the vertical flux-density scales. The data are from Arecibo at 5.5 GHz. Use the pulse width and a light-travel time argument to estimate the size of the emitting region. The pulsar is 2 kpc away. The Crab pulsar is embedded in the Crab Nebula, whose flux density is about 900 Jy. (The data are from a paper, *Nanosecond radio bursts from strong plasma turbulence in the Crab pulsar*, Hankins et al. 2003, Nature, 422, 141). If the true electron energy is \( \gamma m_e c^2 \) with \( \gamma = 10^5 \), estimate how many particles must be radiating coherently to account for the observed radiation.

4. Suppose that a radio telescope is built having 1 km\(^2\) collecting area with 80% aperture efficiency and 20 K system temperature. A transmitter at distance \( D \) radiates 1 Mwatt into a 0.1 Hz bandwidth and into a 1 arc min beam (main lobe; ignore sidelines).

   (a) What is the the maximum \( D \) for which the signal could plausibly be detected?

   (b) Turbulence in the interstellar medium causes spectral broadening by scattering of radio waves from moving plasma fluctuations. Suppose the spectral broadening scales with distance as \( \Delta \nu = 0.1 \text{ Hz}(D/1\text{ kpc}) \). How does your answer to part (a) change?

5. Derive the Ruze formula that gives the reduction factor for the efficiency of a reflector. Do so by first considering a perfect reflector that brings all reflected rays to a perfect focus (i.e. in phase). Now approximate surface imperfections as a set of \( N \gg 1 \) equal-area patches. Each patch is displaced from the local normal of the reflector by a constant amount (over the patch). The distribution of displacements has a Gaussian distribution with zero mean and variance \( \sigma^2_p \). What is the relationship of the variance in phase \( \sigma^2_\phi \) to \( \sigma^2_p \)? Show that the Ruze factor is

\[
\eta = e^{-\sigma^2_\phi} = e^{-(4\pi \sigma_p / \lambda)^2}.
\]

You may wish to use the fact that the average of a complex exponential \( e^{i\phi} \) over a zero-mean Gaussian distribution for the phase \( \phi \) is \( e^{-\sigma^2_\phi / 2} \). Notes on the course web site under Lecture 8 may also be useful.

6. Suppose a gamma-ray burst that emits \( 10^{50} \) erg into \( \gamma \)-rays is accompanied by a radio burst of duration 1 sec that has \( 10^{-5} \) of the gamma-ray energy. Assume the radio burst has a flat spectrum that is 1 GHz wide centered on 1.5 GHz. Also assume the radio emission is beamed into 1 deg\(^2\). If the source is at a distance of 1 Gpc, what would be the peak flux density of the radio burst?

7. In this problem you are to compare radiometer and confusion noise. An expression for the one-sigma confusion noise is given in the handout *Fundamentals of Radiometry*. Consider a multifrequency survey in which you use bandwidths \( B \) at different frequencies; the same antenna is used at all frequencies and it has beam solid angle \( \Omega_A \) at 1 GHz corresponding to a FWHM of 10 arc min. At what integration time does the radiometer noise equal the confusion noise? Use \( B = 10 \) MHz at RF = 0.4 GHz, \( B = 50 \) MHz at RF = 1.5 GHz, \( B = 500 \) MHz at RF = 5 GHz, and \( B = 1 \) GHz at RF = 10 GHz. Does it make sense to integrate longer or shorter than this time? What is the optimal integration time?
8. Consider two different ways of characterizing telescope performance. One is “point source sensitivity,” which is simply $A_e/T_{sys}$ and is proportional to the system equivalent flux density, $\text{SEFD} = T_{sys}/G$. Another is “survey speed,” which is a measure of the rate at which the sky can be surveyed to a certain depth. By considering the volume that is surveyed if radio sources are standard candles, derive the expression

$$SS = \Omega (A_e/T_{sys})^2 B,$$

where $\Omega$ is the solid angle sampled by an antenna, $A_e$ is the effective area, $T_{sys}$ is the system temperature, and $B$ is the bandwidth. $\Omega \sim (\lambda/D)^2$. 

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Figure 2: Intensity, polarization and time structure of nanospheres. The upper panel shows details of pulse 3 in Fig. 1, plotted with 2-ns resolution. Sections of 100-ns duration showing the polarized flux from six of the nanospheres, labelled a-f, are plotted below with left circular polarization upward, and right circular polarization downward, also with 2-ns resolution. The r.m.s. noise level is 18 Jy. The temporal interstellar scattering broadening, $\tau_{\text{IS}}$, due to multipath propagation through the turbulent Crab nebula and the interstellar medium, is negligible; scaling our own previously measured 0.33 and 1.4 GHz values as $\nu^{-4.4}$ (Kolmogorov spectrum), we expect $\tau_{\text{IS}} = 1.9$ ns at 5.5 GHz, and 0.2 ns at 8.6 GHz. These values are roughly consistent with interstellar scattering model predictions (J. M. Cordes and T. J. W. Lazio, personal communication) of $\tau_{\text{IS}} = 0.5$ and 0.05 ns for $\nu = 5.5$ and 8.6 GHz, respectively.