Ray Tracing and Aberrations

Astronomy 6525

Lecture 02

Outline

- Ray Tracing
- Laws of Geometrical Optics
- Sketching Rules
  - Thin Lenses, Multiple elements, Mirrors
- Meridional Ray Trace
  - Examples, Special Rays, Limits
- Aspheric surfaces
- Optical Aberrations
  - On-axis and off-axis
Ray Tracing

- Ray Tracing -
  - Allows study of the performance of an optical system via geometrical optics.

- Different types of rays:
  - Paraxial - rays very close to the optical axis
  - Marginal - ray at the edge of the entrance pupil
  - Meridional - rays restricted to a plane containing the optical axis
  - Skew - rays traveling in any direction

Laws of Geometrical Optics

- Law of Transmission
  - Light travels in a straight line in a region of constant refractive index

- Law of Reflection
  - The angle of incidence equals the angle of reflection.

- Snell’s law
  - \( n_i \sin \theta_i = n_t \sin \theta_t \)
Total Internal Reflection

When \( n_i > n_t \) then we can have

\[
\sin \theta_i = \frac{n_i}{n_t} \sin \theta_t > 1
\]

Which can’t happen \( \Rightarrow \) **Total Internal Reflection**

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Lens Sketching Rules

1. Place object to the left of an optical system and trace rays from left to right
2. A light ray parallel to the optical axis will pass through the focus of the lens (red line).
3. A light ray through the focal point will be refracted parallel to the optical axis (blue line).
4. A light ray through the center of the lens is undeviated (green line).
Tracing Multiple Elements

Concave Lens  |  Convex Lens

- upright, virtual image
- inverted, real image

Combinations or multiple elements:
- An image of an element becomes the object for the next element.

Mirror Sketching Rules

1. Place object to the left of an optical system and trace rays from left to right.
2. A ray parallel to the optical axis will be reflected through the focus of the lens (red line).
3. A ray through the focus will be reflected parallel to the optical axis (blue line).
4. A ray reflected at the vertex makes an equal angle w.r.t. the optical axis (surface normal) as the incident ray (green line).
5. A light ray through the center of the curvature is reflected back on itself (maroon line).
Meridional Ray Trace

Meridional ray specified by:
\( U = \text{slope angle} \)
\( Q = \perp \text{distance from vertex} \)

Given these
\( I, U, Q \) = incident ray
\( I', U', Q' \) = reflected/refracted ray

Want these (equivalent sketch to above)

\[ Q = r \sin I + r \sin U \]
\[ \sin I = \frac{Q}{r} - \sin U \]  \hspace{1cm} \text{Eq. (1)}

Snell’s law:
\[ n \sin I = n' \sin I' \]  \hspace{1cm} \text{Eq. (2)}

\[ \angle PCA: \quad \text{PCA} = I + U = I' + U' \]
\[ U' = I + U - I' \]  \hspace{1cm} \text{Eq. (3)}

Primed version of equation 1:
\[ Q' = r \left[ \sin I' + \sin U' \right] \]  \hspace{1cm} \text{Eq. (4)}

Using equations 1-4 we can determine \( U' \) & \( Q' \) from \( U \) & \( Q \) of the incident ray and the surface data \( r, n, \) and \( n' \).
Planar Surface

The case of a planar surface must be considered separately.

\[ I = U \]

\[ Q \]

\[ Y \]

\[ Z \]

\[ \begin{align*}
Y &= \frac{Q}{\cos U} = \frac{Q'}{\cos U'} \\
\Rightarrow \quad Q' &= \frac{Q \cos U'}{\cos U}
\end{align*} \]

This plus Snell’s law

\[ \sin U' = \frac{n'}{n} \sin U \]

Define \( Q' \) and \( U' \) for a plane.

\[ C = \frac{1}{r} \]

\( = \) surface curvature

If \( C = 0 \), use plane surface equations.

Transferring between surfaces

Now:

\[ Q_2 = Q'_1 + d \sin U'_1 \quad \text{and} \quad U_2 = U'_1 \]

Thus we have \( Q \) and \( U \) for the next surface, etc.

Also need
1) Start-up: transfer from “near-field” or “far-field” object
2) Finish: transfer to focal plane

See Kinglake, pages 24-29.
Examples: Out of Focus

- Slope tells how far out
- Quadrant tells direction (inside or outside)

- Usually the distance, $L$, from the last vertex to the intersection of the ray with $x$-axis is plotted vs. $Y$.
- Thus the focus is automatically “taken out”.

Examples: Spherical Aberration

- Get quadratic term for spherical aberration
Special Rays

- **Paraxial Ray**
  - A ray that always stays near the optical axis
  - small \( U \)'s and \( I \)'s \( \Rightarrow \) \( \sin I \rightarrow I \), \( \cos I \rightarrow 1 \), etc. for \( U \)
  - Called First Order or Gaussian Optics
  - Linear theory - the ray traces equations now have algebraic solutions.

- **Marginal Ray**
  - A ray passing through the edge of the first element
  - Spherical aberration can be evaluated with one paraxial and on marginal ray.
  - \( LA = \text{spherical aberration} = L_{\text{marg}} - L_{\text{paraxial}} \)

**Limit: \( U = 0 \), Paraxial ray**

\[
\begin{align*}
L' &= \text{distance of image from vertex} \\
L' &= \text{distance of image from vertex} \\
\end{align*}
\]

We have \( (\sin I \sim I \text{, etc}): \)

\[
\begin{align*}
I &= \frac{Q}{r} \quad & n &= -n' \\
\Rightarrow \quad I &= -I' \quad & U' &= 2I = -2I' = 2Q/r \\
& \quad & Q' &= r(I' + U') = r(-I + 2I) = rI \\
& \quad & L' &= \frac{Q'}{U'} = r\frac{I' + U'}{U'} = r\left(1 + \frac{I'}{-2I'}\right) = \frac{r}{2} \\
\end{align*}
\]
Limit: $U = 0$

We have:

\[ \sin I = \frac{Q}{r} \quad \text{and} \quad n = -n' \]

\[ \Rightarrow I = -I' \quad \text{and} \quad U' = 2I = -2I' \]

\[ \& \quad Q' = r(\sin I' + \sin U') \]

\[ \Rightarrow L' = \frac{Q'}{\sin U'} = r \left( 1 - \frac{\sin I}{\sin 2I} \right) = r \left( 1 - \frac{1}{2} \frac{1}{\sqrt{1 - (Q/r)^2}} \right) \]

Notice that $L'$ depends on $Q$, the height above the optical axis.

Aspheric Surfaces

- An aspheric surface can be expressed as a departure from a sphere of curvature, $c = 1/r$

\[ X = \frac{cY^2}{1 + \left(1 - c^2Y^2\right)^{1/2}} + a_4Y^4 + a_6Y^6 + \cdots \]

- If the surface is a conic section then we have

\[ (1 - e^2)X^2 - 2rX + Y^2 = 0 \quad \text{or} \quad X = \frac{cY^2}{1 + \left[1 - c^2Y^2\left(1 - e^2\right)\right]^{1/2}} \]

- Where $c$ is the vertex curvature and $e$ is the eccentricity, $K = -e^2$ is the conic constant.
Conic Surfaces

<table>
<thead>
<tr>
<th>Surface</th>
<th>Eccentricity</th>
<th>Conic Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hyperbola</td>
<td>&gt; 1</td>
<td>&lt; -1</td>
</tr>
<tr>
<td>Parabola</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>Prolate spheroid (small end of ellipse)</td>
<td>&lt; 1</td>
<td>-1 &lt; K &lt; 0</td>
</tr>
<tr>
<td>Sphere</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Oblate spheroid (side of ellipse)</td>
<td>--</td>
<td>&gt; 1</td>
</tr>
</tbody>
</table>

Ellipse: \( e = \sqrt{a^2 - b^2} \) / a  
Hyperbola: \( e = \sqrt{a^2 + b^2} \) / a

\( a = \) semi-major axis,  
\( b = \) semi-minor axis

For some plots and info on optical application of conics see  
http://www.telescope-optics.net/conics_and_aberrations.htm

Aberrations

- On-axis:
  - Chromatic
  - Spherical
- Off-axis
  - Coma
  - Astigmatism
  - Field curvature
  - Distortion
Chromatic Aberration

- Spherical and chromatic aberrations are the only “on-axis” aberrations.
- Chromatic aberration occurs for lenses only.

\[
\frac{1}{f} = (n - 1) \left[ \frac{1}{r_1} - \frac{1}{r_2} \right]
\]

\[n = n(\lambda)\]

Optical Dispersion

- Note: dispersion and absorption are not just inconvenient, but necessary consequences of refractive index and also necessarily larger for larger refractive index (c.f. Drude-Lorentz model and Kramers-Kronig relations)
- A “perfect” glass with \(n\)=constant and no absorption does not satisfy Maxwell’s equations
Longitudinal Spherical Aberration

- Spherical Aberration: Light rays striking the entrance aperture different heights but parallel to the optical axis focus at different places.
- Longitudinal Aberration (LA): \( LA = L - L_{\text{paraxial}} \)

Transverse Spherical Aberration

- \( TA = \) distance of ray from the axis
Coma

The effective focal lengths and transverse magnifications differ for rays transversing off-axis regions of the lens.

Coma \propto y^2 \theta

Coma Diagram

- For an oblique bundle of rays, coma occurs when the intersection of the rays is not symmetrical, that is, shifted w.r.t. the axis of the bundle.
- Focal length and magnification are different for each “ring” on lens.
- Want paraxial and marginal magnifications to be the same.
Abbe Sine Condition

- For an optical system to be free of coma, it must obey the Abbe sine condition which for a very distant object is:

\[
\frac{h}{\sin U'} = C
\]

- \( h \) = height of ray before it enters the system
- \( U' \) = angle between ray and optical axis as it travels towards focus
- \( C \) = constant

Astigmatism

The foci in the tangential (meridional) and sagittal planes are at different locations.
Field Curvature

- With a finite aperture the image plane is a curve.
- Also called Petzval field curvature

Distortion

- Happens if the transverse magnification is a function of the off-axis image distance.
- Each point may be sharply focused but the image is distorted.
- Introduction of a stop can cause distortion.
Expanding beyond the linear approximation gives the third-order Seidel aberration terms. The angular aberrations are:

\[ AA = a_1 \frac{y^3}{R^3} + a_2 \frac{y^2 \theta}{R^2} + a_3 \frac{y \theta^2}{R} + a_4 \frac{\theta^3}{R} + a_5 \theta^3 \]

- **\( a_i \)** = constants
- **\( R \)** = radius of curvature
- **\( y \)** = height of ray
- **\( \theta \)** = angle of incidence of rays from object at infinity

**Notes:**
- **\( AA \)** = angular aberration (e.g., arcsec or radians)
- **\( \theta \)** = angle of incidence of rays from object at infinity
- **\( \theta_{\text{max}} \)** = angle of incidence of rays at which aberration is maximum
- **\( AA_{\text{max}} \)** = maximum angular aberration

Taking **\( R = f_n y \)** then:

\[ AA_{\text{max}} = a_1 \frac{y^3}{f_n^3} + a_2 \frac{y^2 \theta}{f_n^2} + a_3 \frac{y \theta^2}{f_n} + a_4 \frac{\theta^2}{f_n} + a_5 \theta^3 \]

- **\( f_n \)** = f-number
- **\( D \)** = aperture diameter

**Note:** the “faster” the optical system, the greater the aberrations!