Fabry-Perot Interferometers

Astronomy 6525

Literature:
• C.R. Kitchin, “Astrophysical Techniques”
• Born & Wolf, “Principles of Optics”
Theory: 1

Fabry-Perots are best thought of as resonant cavities formed between two flat, parallel highly reflecting mirrors. The pair of mirrors is called an *etalon.*

Let:
- \( d \) = spacing between the mirrors
- \( \tau \) = amplitude transmissivity
- \( \rho \) = amplitude reflectivity

For a collimated beam with amplitude, \( A \), incident at angle \( \theta \), the phase shift (difference) between successive transmitted rays is:

\[
\delta = \frac{2\pi}{\lambda} 2d \cdot \cos \theta
\]
Here we ignored the phase shift on reflection (this just acts like a change in the cavity size) and assumed that the index of refraction between the mirrors is 1.

The reflection phase shift can be included by letting:
\[ d \rightarrow d + \Delta d \]

where:
\[ \Delta d = -\frac{\phi \cdot \lambda}{2\pi} \cos \theta \]

and \( \phi \) is the reflection phase shift (excepting the first ray). The complex amplitude for the \( n^{th} \) transmitted ray is:
\[ \psi_n = A \tau^2 \rho^{2(n-1)} e^{i(\omega t -(n-1)\delta)} \]

and the total amplitude is:
\[ \psi = \sum_{n=1}^{\infty} \psi_n = A \tau^2 e^{i\omega t} / (1 - \rho^2 e^{-i\delta}) \]
Theory: 3 (Airy Function)

So that the transmitted intensity is:

\[ I \equiv \psi^*\psi = A^2\tau^4/(1 - 2\rho^2\cos\delta + \rho^4) \]

Rewriting in terms of the mirror intensity coefficients:

\[ t \equiv \tau^2 \quad \text{and} \quad r \equiv \rho^2 \]

The transmitted intensity of the etalon is:

\[ \frac{I(\delta)}{I_0} = \frac{T_{\text{max}}}{1 + \left[\frac{4r}{(1 - r)^2}\right] \cdot \sin^2(\delta/2)} \]

where:

\[ T_{\text{max}} \equiv \frac{t^2}{(1 - r)^2} \]

The form above is that of the Airy function. The incident intensity is given by:

\[ I_0 = A^2 \]
Theory: 4 (Transmission Peaks)

In the limit of no absorption:

\[ t = 1 - r \text{ and } T_{\text{max}} = 1 \]

The Airy function has peak when the phase shift, \( \delta \), is an integer multiple of \( 2\pi \), e.g.

\[ \delta = 2\pi m \quad \text{m = 0, 1, 2, ... = order of etalon} \]

where:

\[ m = \frac{2d}{\lambda} \]

for normal incidence.
Theory: 5 (Peak Width)

Set:

\[
\frac{I(\delta)}{I_0} = \frac{T_{\text{max}}}{1 + \left[\frac{4r}{(1 - r)^2}\right] \cdot \sin^2(\delta/2)} = \frac{T_{\text{max}}}{2}
\]

Solve for \(\delta\), resulting in \(\delta_{\text{HWHM}}\)

Then, the full-width at half-maximum (FWHM) of a single peak is (phase):

\[
\delta_{\text{FWHM}} = 2 \delta_{\text{HWHM}} = 4 \sin^{-1}\left(\frac{(1 - r)}{2\sqrt{r}}\right) \approx 2\frac{(1 - r)}{\sqrt{r}}
\]
The spacing between transmission peaks (in units of wavelengths) is called the **free spectral range**. We have:

\[ m\lambda = (m - 1) \cdot (\lambda + \Delta\lambda_{FSR}) \]

\[ \Rightarrow \Delta\lambda_{FSR} = \frac{\lambda}{m} \]

Thus:

\[ \Delta\lambda_{FSR} = \frac{\lambda}{m} = \frac{\lambda^2}{2d} \]
Theory: 7 (Reflectivity Finesse)

The finesse, $F$, of an FPI is the ratio of the distance between peaks, to their FWHM.

$$F \equiv \frac{2\pi}{\delta_{\text{FWHM}}} \approx \pi \sqrt{r/(1 - r)}$$

This approximation is accurate to within 1% for $r > 0.65$.

For normal incidence, we can compute the single peak width, from the definition of $\delta$ $(=4\pi d/\lambda)$:

$$\Delta\delta = \frac{4\pi d \cdot \Delta\lambda_{\text{FWHM}}}{\lambda^2} = \delta_{\text{FWHM}}$$

hence:

$$\Delta\lambda_{\text{FWHM}} \approx \frac{\lambda^2}{2d} \cdot \frac{1 - r}{\pi \sqrt{r}}$$

so that:

$$F = \frac{\lambda^2}{2d \cdot \Delta\lambda_{\text{FWHM}}} = \frac{\lambda}{m \cdot \Delta\lambda_{\text{FWHM}}}$$

The finesse along with the transmission, is the primary figure of merit for an FPI.
Theory: 8 (Resolving Power)

The resolving power:

\[ R_{FPI} \equiv \frac{\lambda}{\Delta \lambda_{\text{FWHM}}} \]

\[ = m \cdot \left( \frac{\lambda}{m \Delta \lambda_{\text{FWHM}}} \right) = m \cdot F \]

Since the resolving power in an interferometer is determined by the largest difference between interfering rays, F can be thought of as a path length multiplier, and is roughly the number of reflections an average photon makes before being transmitted through the system.
Theory: 9 (Resolution Limitation)

Both the bandwidth ($\propto 1/(\text{resolving power})$) and the distance between peaks (FSR) scale inversely with plate separation $d$.

Their ratio is a constant.

In other words, when the resolution gets very large, the free spectral range gets small, and therefore:

At high resolution the orders of an FPI will tend to overlap.
Another figure of merit for the FPI is the contrast, which is the ratio of the maximum to minimum transmission:

\[ C \equiv \frac{T_{\text{max}}}{T_{\text{min}}} = \frac{(1 + r)^2}{(1 - r)^2} = 1 + \left(\frac{2F}{\pi}\right)^2 \]

For a \( F > 15 \), \( C > 100 \), which is adequate.

\( C \) is a strong function of \( F \): \( F > 45 \Rightarrow C > 1000 \)
Limits to Performance: I

1. Mirror Absorption

If $a$ is the single pass intensity absorption coefficient, then the maximum transmitted intensity is:

$$T_{\text{max}} = \left\{ \frac{t}{(1 - r)} \right\}^2$$

$$= \{(1 - r - a)/(1 - r)\}^2 = \{1 - a / (1 - r)\}^2$$

The reflectivity finesse is independent of the absorption coefficient. Note that for small $a \ (\leq 2\%)$, the transmission is $1 - aF$, since $F \sim$ the number of reflections. For $\frac{aF}{\pi \sqrt{r}} \ll 1$:

$$T_{\text{max}} \sim 1 - 2aF / (\pi \sqrt{r})$$

Since $F = \pi \sqrt{r} / (1-r)$
Limits to Performance: II

Mirror Absorption

Transmission of an FPI as a function of \( a = 1 - t - r \). Typically \( a \sim 1\% \), for which a good FPI will have \( F \sim 30 \) to 50 \((r = 0.90 \rightarrow 0.94)\)

It is important to keep \( T \) close to unity, since large \( F \) is not useful.

Bradford 2001 Phd Thesis, Cornell

Figure 3.5: Transmission of an FPI etalon as a function of the single-pass absorption coefficient \( a = 1 - t - r \) of a single mesh. For the Buckbee-Mears meshes we use, \( a \sim 0.01 \), and the lower finesse cavities result in substantially higher transmission. For this reason, a triple FPI system such as SPIFI results in a better transmission \( F \) than a double of the same resolving power.
Limits to Performance: III

2. Etalon Tip - Walk Off
Tipping the etalon causes beams to “walk” off the mirrors. The beam will walk a distance \( w \) before it is transmitted (after \( F \) reflections) given by:

\[
w = F \cdot d \cdot \sin(\theta_{\text{Tip}}) = R \cdot \lambda / 2 \cdot \theta_{\text{sky}} (D_p / D_C)
\]

Things get bad when the central ray has traveled a distance equal to the beam radius across the etalon, the constructive interference is significantly reduced:

\[
D_C / 2 \approx F_{\text{Tip}} \cdot d \cdot \sin(\theta_{\text{Tip}}) \approx F_{\text{Tip}} \cdot d \cdot \theta_{\text{Tip}} \Rightarrow F_{\text{Tip}} \approx D_C / (2d \cdot \theta_{\text{Tip}})
\]

where \( D_C \) is the beam diameter.
**Etalon Tip**

Notice that tipping the etalon shifts the central wavelength of transmission to shorter wavelength:

\[
\begin{align*}
\theta_{\text{Tip}} &= 0: \quad 2d/\lambda_o = m \\
\theta_{\text{Tip}} \text{ small:} \quad 2d/\lambda_1(\cos\theta) = m \\
&\Rightarrow 2d/\lambda_1(1 - \theta^2/2) = m \\
&\Rightarrow \lambda_1 - \lambda_o \equiv \Delta\lambda = -\lambda_o \theta^2_{\text{Tip}}/2 \\
&\Rightarrow \Delta\lambda = -\lambda_o \theta^2_{\text{Tip}}/2 \quad \text{(blue shift)}
\end{align*}
\]
3. Mirror Parallelism
If one mirror is tipped with respect to the other at an angle $\theta$, then the problem is much more severe than a mere tipping of the entire etalon. Not only does the reflected angel grow with each reflection, but the cavity resonates at a different wavelength at different positions!!
Mirror Parallelism

To make an estimate of this effect, let's assume resonance at $\lambda_o$ at one end of the etalon, and at $\lambda_1$ at the other. The maximum resolution of the etalon is then:

$$R_{\text{max}} = \frac{(\lambda_o + \lambda_1)}{2(\lambda_o - \lambda_1)} = F_{\text{Par}} \cdot m$$

Now, if $\Delta$ is the gap difference, i.e. $\Delta = D_C \sin \theta_{\text{par}}$ (where $D_C=$beam diameter, $\theta_{\text{par}}=$deviation from parallelism), then:

$$(\lambda_o - \lambda_1) = 2d/m - 2(d + \Delta)/m = -2\Delta/m$$

or, letting $(\lambda_o + \lambda_1)/2 \rightarrow \lambda$, then:

$$F_{\text{par}} \cdot m = m\lambda/(2\Delta) \Rightarrow F_{\text{Par}} = \lambda/(2\Delta)$$

If the mirrors are parallel to $\lambda/n$ then $F_{\text{Par}} = n/2 \Rightarrow$ you need parallelism to better than $\lambda/200$ or so!
4. Aperture Effects

The resonant condition is: \( \frac{2d \cdot \cos \theta}{\lambda} = m \)
- normal ray: \( \frac{2d}{\lambda_0} = m \)
- off-axis ray: \( \frac{2d \cdot \cos \theta_{FP}}{\lambda_1} = m \)

\[ \Rightarrow \frac{2d}{\lambda_0} \approx \frac{2d}{\lambda_1} \left(1 - \frac{\theta_{FP}^2}{2}\right) = m \]
\[ \Rightarrow \lambda_1 - \lambda_0 \equiv -\Delta \lambda = -\lambda_0 \theta_{FP}^2/2 \]
\[ \Rightarrow \Delta \lambda/\lambda_0 \equiv 1/R = \theta_{FP}^2/2 \equiv 1/(F_{\text{aperture}} \cdot m) \]
\[ \Rightarrow F_{\text{aperture}} = R/m = 2\pi/\{m\pi(\theta_{FP}^2)\} = \pi\lambda_0/(d \cdot \Omega_{FP}) \]
\[ \Rightarrow R = 2\pi/\Omega_{FP}, \text{ or } R \cdot \Omega_{FP} = 2\pi \]
Aperture Effects
Now, since etendue is conserved:

\[ A_{FP} \Omega_{FP} = A_{Primary} \Omega_{Primary} \]
\[ \Rightarrow \Omega_{Primary} \cdot A_{Primary}/A_{FP} \cdot R = 2\pi \]
\[ \Rightarrow R_{\text{max}} = 2\pi{A_{FP}/A_{Primary}}/\Omega_{Primary} \]

5. Diffraction Effects
Diffraction results in a minimum angle within the beam given by:

\[ \theta_{\text{Teldiff}} \sim \lambda/D_{\text{Tel}} \]

Plugging this into our aperture effects equation:

\[ R_{\text{diff}} = 2\pi{A_{FP}/A_{Primary}}/{\pi/4 \cdot (\lambda/D_{\text{Primary}})^2} = 8 \cdot (D_{FP}/\lambda)^2 \]

where: \( \Omega = \pi/4 \cdot (\lambda/D)^2 \), \( D_{FP} \) is the diameter of the beam through the etalon.
6. Surface Defects

Irregularities in the surfaces of the mirrors will cause irregularities in the path lengths for different regions of the cavity. Suppose we have an rms gap variation that is given by $\Delta s$, and is distributed in a Gaussian manner. It can be shown that:

$$F_{\text{surf}} = \frac{\lambda}{((32\ln2)^{1/2}\Delta s)} \approx \frac{\lambda}{(4.7\Delta s)}$$
Effective Finesse and Transmission

Each effect reduces both the FPI transmission, and the finesse. The total effective finesse is given by the inverse quadratic sum of each finesse term above with the reflective finesse (Cooper, J. and Greig, J.R. 1963 J. Sci. Instrum., 40, 433.):

$$\frac{1}{F_{\text{eff}}}^2 = \sum \frac{1}{F_i^2}$$

The peak transmission is reduced, since some of the radiation is absorbed, some of it is forced out of the beam, and some of it interferes destructively. The etalon transmission is reduced by the ratio of the effective finesse to the reflective finesse:

$$T_{\text{FPI}} = T_{\text{max}} \cdot \left\{ \frac{F_{\text{eff}}}{F_{\text{ref}}} \right\}$$

$$= \left[ \frac{(1 - r - a)}{(1 - r)} \right]^2 \cdot \left\{ \frac{F_{\text{eff}}}{F_{\text{ref}}} \right\}$$
Limits to Performance: XI

Optimizing Performance
In a background limited situation, one can characterize the optimal performance of an FPI by the product of the finesse times the transmission:

\[ \text{NEP} \equiv P_{S|S/N=1} \propto T^{-1/2} \cdot R^{-1/2} \propto (F \cdot T)^{-1/2} \]

(where \( P_{S|S/N=1} \) is the signal power from the sky that is equivalent to the background noise)

Therefore, we would wish to maximize the product:

\[ Q \equiv F \cdot T \]

Now:

\[ T_{\text{max}} \approx \{1 - (2aF)/(\pi \sqrt{r})\} \]

\[ \Rightarrow Q \approx \{1 - (2aF)/(\pi \sqrt{r})\} \cdot F \]

\[ \Rightarrow \frac{dQ}{dF} \approx 1 - \frac{(4aF)/(\pi \sqrt{r})}{0} = 0 \]

For example:

\[ F_{\text{optimal}} \approx \frac{\pi}{4a} \approx 80 \text{ for } a = 1\% \]

\[ \Rightarrow r \approx 96\% \]
Applications of Fabry-Perot Interferometers

- Use a Fabry-Perot with high resolution as the spectral element that defines the resolution.
- To filter out unwanted transmission peaks of a Fabry-Perot:
  - Use several Fabry-Perots in series
  - Use additional edge filters and band pass filters
  - Some detectors are not sensitive to wavelengths outside of their range
- The Fabry-Perot that determines the spectral resolution is scanned over a wavelength range, by changing the plate separation to obtain a spectrum at each spatial position on the sky.
  - Resulting data is a 3D data cube:
    - 2 spatial dimensions (from 2D detector array)
    - 3rd dimension is spectral dimension (scanning time)
Example: The UC Berkeley Tandem FPI

**Optical filtration train:**
Tunable high order FPI (HOFPI) is the resolution achieving device
- $n \approx 20$ to $600 \Rightarrow R \approx 1000$ to $36,000$
  (F varied from 30 to 60)
- HOFPI fringes evenly spaced frequency space

**Fixed filters (low-order FPI, LOFPI)**
- Set in 1$^{st}$ to 4$^{th}$ $\Rightarrow R \approx 30$ to $260$
  (optimize F for line of interest)
- One filter for each line of interest
- Up to 6 filters on a wheel

**Salt Filters**
- Act as low frequency pass filters
- Block typically $> 20 \ \mu m$
- Can be put on wheel (specialized)

Transmitted profile is the product of each element of the train.
Examples of Fabry-Perot Instruments
Kuiper Widefield Infrared Camera: KWIC

- KWIC is an Imaging Spectrometer/Photometer
  Tandem FPI's
  1. R > 2000 (up to 10,000): Spectral Lines
  2. R ~ 35, 70, 105: Dust Continuum

- Wide field of view with diffraction limited beams using a
  128 × 128 pixel Si:Sb BIB Array ⇒ 5.8' × 5.8' FOV

- Far-IR (18 μm < λ < 38 μm)
  Lines:  [SIII] 18 and 33 μm
  [SII] 35 μm
  [NeIII] 36 μm
  etc.
  Continuum: T_{\text{planck}} ~ 85 K at 38 μm
Optical Layout For KWIC: I

Bottom View

- Beam enters at left at f/12.6
- Passes through calibration unit
- Gas cell for spectral calibration
- Hot chopped blackbody for flatfielding
- HOFPI in collimated beam to minimize aperture effects on R
- LOFPI in focus – makes parallelizing easy
- Reimaging optics to match f/# to pixel size

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Fabry-Perot Interferometer in the Mid-IR to Submm Wavelength Regime

- In the mid-IR ($\geq 25 \mu m$) and up to submm wavelengths Fabry-Perot “plates” can be made using free-standing metal meshes
- Scanning and fixed Fabry-Perot Interferometers with free-standing metal meshes have been used in many instruments
Fixed Wavelength Filters

Fixed $\lambda$ filters are used to sort the orders of the HOFPI:

• Made from free standing metal mesh
• Stretched onto stainless steel rings
• Push and pull screws to align and fix FPI
• Optically flat to about $\frac{1}{4}$ HeNe wavelength
• Finesse typically 30 to 60, orders between 1 and 5, $\Rightarrow R \sim 30$ to 300
• Peak transmissions from 25 to 90%
• Quite robust, cycled dozens of times over many years


Figure 2.12. New design for the fixed wavelength FPI's. The plate spacing and tilt are set using three pairs of opposing adjustment screws.
Properties of Mesh FPI: Finesse vs. Wavelength

- Worked with a variety of mesh grid spacings and optical transmissions
- Finesse typically scales as $\lambda^2$
- Select mesh to keep $F$ reasonable ensuring good transmission.

\[ \propto \lambda^2 \]
Example of a Fabry-Perot Design: KWIC LOFPI

KWIC LOFPI

- Flex vane based translation stage ensures parallelism
- Mesh rings are magnetic stainless, held in place with magnets
- Rough adjustment via externally accessible fine-adjustment screw
- Scanning PZT for fine adjustment and scanning
- Spacing measured with a capacitance bridge fed back to PZTs, and held to < 14 nM
- Liquid Helium temperature operation
- Tilt PZT’s travel ~ 2 µm

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