Radio Telescopes

The first 50 years: experimentation, discovery and striving for sensitivity + angular resolution, time resolution

Now: Experiments, precision cosmology and astrophysics, a new era of exploration: sensitivity + resolution + high time and frequency resolution
Karl Jansky
Bell Telephone Laboratory 1933
Reber Telescope (1940s)

Figure 1.5: The 85-foot diameter radio telescope used by Frank Drake at the National Radio Astronomy Observatory in Green Bank, West Virginia, for Project Ozma in April 1960. Part of the team assembled for the 40th anniversary of the project, including Drake, standing second from the right.
Nancay Radio Telescope

Molonglo Observatory Synthesis Telescope
Green Bank Telescope

Ryle Synthesis Telescope (Cambridge)
Westerbork Synthesis Radio Telescope

Very Large Array
PSR B0919+06
S. Chatterjee et al. (2000)

μ = 88.5 ± 0.13 mas/yr
π = 0.83 ± 0.13 mas

D = 1.2 kpc
V = 505 km/s
1. Proper Motions of Pulsars

- Interferometry (VLA, MERLIN, VLBA)
  - mas/yr - arcsec/yr
- Timing
  - microsec - ms
- Interstellar Scintillation
  - intensity (t, ν) (minutes, MHz)

Maser emission from water molecules orbiting a central black hole in NGC5258

VLBA
Reflector Optics

Prime focus
Cassegrain focus
Offset Cassegrain
Naysmith
Beam Waveguide
Dual Offset

Reflector Optics: Examples

Prime focus
(CMRT)

Cassegrain focus
(AT)

Offset Cassegrain
(VLA)

Naysmith
(OVRO)

Beam Waveguide
(NRO)

Dual Offset
(GBT)
New Telescopes
1. Beam Patterns and Pixels

- All telescopes have an angular response to radiation
- Optical/IR/X-ray/Gamma-ray: point spread function (PSF)
- Radio: antenna power pattern $P_n(\theta)$
- A single reflector with a single feed antenna at the focus has one pixel

Modern cm-wavelength radio telescopes using unblocked apertures top avoid stray radiation (scattering) and to minimize side lobes (satellite RFI).
Two kinds of “gain”

There are two kinds of gain encountered in radio astronomy:

**Astronomer’s gain:**

\[ G = \frac{A_e}{2k} \]

expressed in K Jy$^{-1}$

**Engineer’s gain:** (related to directivity of the beam pattern)

\[ G = \frac{4\pi}{\Omega_A} \]

which is dimensionless and where

\[ \Omega_A = \int d\Omega P_n(\Omega) \]

Note that \( \Omega_A A_e \equiv \lambda^2 \) so that

\[ G = \frac{4\pi}{\Omega_A} = \frac{4\pi A_e}{\lambda^2} \]
System Equivalent Flux Density (SEFD):

\[
SEFD = \frac{T_{\text{sys}}}{G}
\]

where \( G \) = astronomer’s gain expressed in K Jy\(^{-1}\). The SEFD is simply the system noise expressed as a flux density. A low value is better than a high value.

Antenna Performance: Aperture Efficiency

On axis response: \( A_0 = \eta A \)

Efficiency: \( \eta = \eta_{\text{sf}} \cdot \eta_{\text{bl}} \cdot \eta_s \cdot \eta_t \cdot \eta_{\text{misc}} \)

\( \eta_{\text{sf}} \) = Reflector surface efficiency
Due to imperfections in reflector surface:

Ruze formula:

\( \eta_{\text{sf}} = \exp(-4\pi\sigma^2/\lambda^2) \) e.g., \( \sigma = \lambda/16 \), \( \eta_{\text{sf}} = 0.5 \)

\( \eta_{\text{bl}} \) = Blockage efficiency
Caused by subreflector and its support structure

\( \eta_s \) = Feed spillover efficiency
Fraction of power radiated by feed intercepted by subreflector

\( \eta_t \) = Feed illumination efficiency
Outer parts of reflector illuminated at lower level than inner part

\( \eta_{\text{misc}} \) = Reflector diffraction, feed position phase errors, feed match and loss
1. Surface of ALMA Vertex Antenna

- Surface measurements of DV02 made with holography
- Measured surface rms = 10μm

Primary Beam

\[ l = \sin(\theta), \ D = \text{antenna diameter in wavelengths} \]
\[ \text{dB} = 10\log(\text{power ratio}) = 20\log(\text{voltage ratio}) \]
\[ \text{VLA: } \theta_{3\text{dB}} = 1.02/D, \ \text{First null} = 1.22/D \]
Diffraction effects cause there to be non-zero gain even from behind a telescope.
Figure 3. The simulated beam pattern of an oversized Cassegrain system at 15 GHz. The inset shows the main beam with an expanded scale.
1. Ray tracing vs Diffraction

- Ray tracing has its uses in radio astronomy but diffraction almost always has to be included in a detailed analysis.

- Why different from OIR?
  - Because $\lambda/(\text{optical-element sizes})$ are larger.
Getting Multiple Pixels

1. Use multiple feed antennas (feed clusters)
   e.g. ALFA = Arecibo L-band Feed Array (7 pixels)

2. Use a phased-array feed system
   Difficult, R&D now a major part of SKA efforts

3. Use arrays of antennas as interferometers and synthesize an aperture

Parkes 64 m 13-beam 21 cm “camera”

The HIPASS survey of HI in external galaxies and a search for new pulsars

Measured ~5,000 galaxies and found ~500 pulsars

Receiver Specifications

The multibeam receiver was designed to have high performance for 21 cm line work. This was achieved by compromising performance in other areas such as frequency range and polarisation purity. The following table lists the receiver specifications.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Beam 1</th>
<th>Beams 2-7</th>
<th>Beams 8-13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Range</td>
<td>1.23-1.33 GHz</td>
<td>1.22-1.32 GHz</td>
<td>1.22-1.32 GHz</td>
</tr>
<tr>
<td>2 orthogonal linear</td>
<td></td>
<td>2 orthogonal linear</td>
<td>2 orthogonal linear</td>
</tr>
<tr>
<td>Polarisations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average System Temperature (K) (Elev=55°)</td>
<td>21 K</td>
<td>21 K</td>
<td>21 K</td>
</tr>
<tr>
<td>FWHM beamwidth</td>
<td>14.0 arcmin</td>
<td>14.1 arcmin</td>
<td>14.5 arcmin</td>
</tr>
<tr>
<td>FWHM beam ellipticity (radial)</td>
<td>0.00</td>
<td>0.03</td>
<td>0.06</td>
</tr>
<tr>
<td>Efficiency</td>
<td>1.36 Jy K⁻¹</td>
<td>1.45 Jy K⁻¹</td>
<td>1.72 Jy K⁻¹</td>
</tr>
<tr>
<td>Average System Temperature (Jy) (Elev=55°)</td>
<td>29 Jy</td>
<td>30 Jy</td>
<td>36 Jy</td>
</tr>
<tr>
<td>Average Cal Temperature (Jy)</td>
<td>1.6 Jy</td>
<td>1.8 Jy</td>
<td>2.0 Jy</td>
</tr>
<tr>
<td>Coma lobe</td>
<td>none</td>
<td>-17 dB</td>
<td>-14 dB</td>
</tr>
</tbody>
</table>
1. Innovations in Radio Telescopes

• Where needed?
  • High angular resolution with high sensitivity
  • Wide overall field of view for
    – Fast surveys of the sky (large numbers of objects)
    – Time variable sources (bursts)

• Challenges:
  • Building collecting area cheaply
  • Exploiting Moore’s law in digital electronics
  • i.e. steel/aluminum vs. silicon/germanium
Phased Array Feeds

- CST simulation study of an array

Focal L-band Array for the GBT (FLAG)

- 19 dual polarized elements. Cryogenic PAF system
- $T_{sys} \sim 20$ K; Aperture efficiency $\sim 75$ to 80 \%; $\frac{T_{sys}}{T_{obs}} \sim 25$ K
- 7 beams; spacing 0.5 FWHM to 1 FWHM
- Frequency coverage – 1300 to 1800 MHz
- Backend for processing signals (beamforming, calibration etc)

FLAG: first prototype (BYU)

- Prototype is a 19 element single polarized array. Freq $\sim 1600$ MHz
Points

• Beamwidths: $\theta \approx \lambda / D$
  – Applies to all detectors of wavelike phenomena
  – Optical/IR telescopes, radio telescopes, our eyes
• Effective temperatures (pretend radiation is blackbody in the Rayleigh-Jeans regime)
• Unit for flux density: the Jansky, used to characterize radio sources
• Radiometer noise: analogous quantity to readout noise + photon counting noise in CCDs (with different statistics)

Nomenclature

• Antenna beam pattern = point spread function (PSF)
  – Beam FWHM = full width at half maximum
  – Beam solid angle $\sim (\pi/4) \text{FWHM}^2$
  – Beam = diffraction effect
• For aperture of size $D$, radiation is diffracted into an angle $\lambda/D$
• The solid angle is then $\sim (\pi/4) (\lambda/D)^2$
  – Sidelobes of the beam pattern
• Radiometry and radiometer equation; radiometer noise; radio source confusion
• Effective temperature of radiation
• System temperature of radiometer
• Heterodyned receiver system, local oscillator, sidebands, baseband
Effective Temperatures

- Radio telescope measurements can be calibrated into temperature units
- Examples:
  - Cosmic microwave background: 2.7 K it is a bb!
  - Clouds in the interstellar medium: HI, HII
    - HI: temperatures 10s of K up to ~ 10^3 K
    - HII: 5000 – 12000 K
  - Sun: 6000K at some λ, 10^6 K at others ???
  - Interstellar masers: up to ~ 10^14 K
  - Pulsars (rotating neutron stars): up to 10^41 K !!!

Flux Densities

- The Jansky unit of flux density was invented in the 1960s and was chosen to be ~ 1 for radio sources detectable then.
- Today (better receivers) we can detect ~ 10^{-6} Jy sources
- Energy in 1 sec in a 1 Hz bandwidth through a 1 cm\(^2\) area:
  - \(\Delta E = F_v \Delta t \Delta v \Delta A = 10^{-23} \) erg or \(10^{-26}\) Joule
- All the energy collected by all the radio telescopes ever ~ 1 erg.
**Extended Sources**

- Definition: Source much larger than antenna power pattern main lobe: $\Omega_s >> \Omega_A$
- Then the measured power is $P_v = kT \Delta v$
- So what?
  - The measured power is independent of the telescope size
  - Why is this useful?

**Mixers**

- Heterodyned systems use mixers
- A mixer is simply a multiplier
- Refers specifically to the product of an RF signal selected by the antenna and a monochromatic “local oscillator” (LO) signal
- Purpose:
  - Shift the RF signal in frequency for easier processing
  - Can shift different RFs to a common “intermediate frequency” (IF) to simplify electronics
    - AM radio
  - Shifting an an IF < RF is easier to digitize
- Analog: a nonlinear device: diodes
- Digital: exact to within number of bits of precision + accuracy of ADC = analog-to-digital converter
Mixers

- Based on identities
  \[2 \cos A \cos B = \cos A + B + \cos A - B\]
  \[2 \cos A \sin B = \sin A + B - \sin A - B\]
- Easier:
  \[e^{iA} e^{iB} = e^{i(A+B)}\]
  \[\Rightarrow\]
  \[\left[ e^{iA} + e^{-iA} \right] \times \left[ e^{iB} + e^{-iB} \right] = e^{i(A+B)} + e^{-i(A-B)} + \text{c.c.}\]

- Shift operator in frequency space
- Let \(A\) = Fourier representation of RF signal
- Let \(B\) = monochromatic signal (local oscillator)
- Multiplication \(\rightarrow\) lower and upper sidebands
- Often one of the sidebands is removed by filtering
- Implemented with analog devices or digitally

A simple heterodyned system

- [Diagram of a simple heterodyned system with RF, LNA, IF, Bandpass Filter, LO, and IF bandwidth \(\Delta v_{IF}\) centered on upper or lower sideband]
RF spectrum selected by feed antenna + LNA

Spectrum after mixer

IF spectrum after filter

IF spectrum selected by feed antenna + LNA

\[ \nu_{RF} = \nu_{RF} \pm \nu_{LO} \]

Lower sideband

See NRAO WEB site www.nrao.edu

Filters (GHz)

<table>
<thead>
<tr>
<th>Frequency Range</th>
<th>Filters</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15-2.00</td>
<td>To Analog Filter 900,200MHz R detectors, DCF 180Hz sample Spectrometer</td>
</tr>
<tr>
<td>0.5-1.0</td>
<td>To VL</td>
</tr>
<tr>
<td>0.15-0.55</td>
<td>To Analog 50, 12.5 kHz detectors, 100 MHz Spectrometer Spec.Prox</td>
</tr>
</tbody>
</table>

IF3 = LO3 - LO2 + IF1
LO3 = Fsky + IF1 (Fsky < 11 GHz)
LO1 = Fsky - IF1 (Fsky < 11 GHz)
LO2 = IF1 + LO3 - IF3

Simplified GBT LO/IF system
Spectrum

RF spectrum selected by feed antenna + LNA

IF spectrum

Parts

Square-Law Detector

Lowpass Filter

Lowpass Filter

Bandpass Filter

ADC

LNA
The time-integrated output has fluctuations given by the radiometer equation

\[
\frac{\sigma_T}{\langle I \rangle} = \frac{1}{\sqrt{\Delta \nu T}}
\]

The radiometer equation is a consequence of summing an ensemble of sinusoids with random phases and the central limit theorem.

## 1. Radiometry

- Radio telescopes are used as radiometers to measure power incident on an antenna
  - A single reflector antenna + single “feed” antenna at the focal point = a single pixel
  - Significant (Nobel prize) single-pixel science:
    - Discovery of the Cosmic Microwave Background
    - Discovery of Pulsars
    - Indirect detection of gravitational waves from a pulsar binary
- Spectroscopy = frequency resolved radiometry
- Sensitivity of a radiometer is characterized in terms of the system temperature:
  - \( T_{sys} = T_{sky} + T_{receiver} + T_{spillover} + \ldots \)
- \( \mu \) wavelengths: Galactic noise from synchrotron radiation dominates
- \( \mathrm{cm} \) wavelengths: receiver + thermal Galactic noise
- \( \mathrm{mm} \) wavelengths: receiver + atmospheric noise
Figure 1. Radio wave microware window.

Figure 2. Terrestrial microwave window.
In contrast to optical and infrared astronomy where it is natural to think of photon detection, radio astronomy generally involves signals formed from large numbers of photons that are, hence, well within the classical regime. Radio signals from natural sources are inherently random, and this randomness determines (i.e. limits), in part, the ability of a radio telescope to discriminate individual radio sources.

A single-antenna radio telescope with standard receiver is a radiometer that measures the power level of the radiation at the frequency $\nu$, in the polarization, and in the solid angle to which the telescope is sensitive. The ability of the telescope to identify an individual radio source depends on:
1. **Radiometer noise:** noise produced by the sky background and by the telescope itself. The sky background consists of an isotropic 3 Kelvin ‘cosmic’ background (the blackbody radiation left over from the big bang); a broadly distributed synchrotron background from galactic cosmic ray electrons; and a thermal background from plasma in the galactic disk. The relative contributions of these backgrounds and instrumental noise depend strongly on radio frequency.

2. **Radio source ‘confusion’:** radiometer variations caused by the ensemble of sources that are in the telescope beam at any instant. For a fixed antenna, as the Earth rotates, individual sources pass through the beam and the number of sources in the beam is a Poisson random variable; thus the radiometer output varies with time. To single out any given source, its flux density must be greater than the root-mean-square variation due to confusion.

3. **Gain variations:** All real world amplifiers and detectors have output levels that vary with time due to various macroscopic (e.g. temperature) and microscopic (1/f noise) causes.

4. **Radio frequency interference (RFI):** Sources of RFI range from the ridiculous to the sublime, being caused by car ignitions; arcing in motors and relays (e.g. the Space Science Bldg. elevator); lightning; aircraft and police transmissions; satellites. RFI is noticeably less at night and on weekends, as well as at remote locations and, generally speaking, at higher frequencies \( f \geq 30 \) GHz.

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**Blackbody Radiation.** Radiation from a blackbody source is described by a brightness distribution (a.k.a. specific intensity; units = energy/time/area/ frequency/solid angle)

\[
B_{\nu} = \frac{2\pi c^2}{c^2 + \nu^4} \frac{1}{e^{\frac{h\nu}{kT}} - 1}
\]

(1)

where \( h = \) Planck’s constant, \( \nu = \) frequency, \( c = \) speed of light, \( k = \) Boltzmann’s constant, and \( T = \) temperature. At radio frequencies, \( h \nu \ll kT \), so expanding the exponential into a Taylor series of two terms, we get the Rayleigh-Jeans approximation

\[
B_{\nu} = \frac{2kT}{\lambda^2}
\]

(2)

where \( \lambda \equiv c/\nu \). It is therefore conventional to refer to radiation intensities in terms of an effective radiation temperature. This is done even in cases where the radiation mechanism is known to differ from blackbody radiation.

\[
T_b = \frac{\lambda^2 B_{\nu}}{2k} \approx \frac{\lambda^2 F_{\nu}}{2k \Omega_s}
\]

Brightness temperature

\[
\Omega_s \approx \pi \theta_s^2 / 4
\]

\[
\theta_s = \text{angular diameter}
\]
1. What is the meaning of $T_b$?

- **Thermal sources:**
  - In some conditions $T_b$ is closely related to the physical temperature in the source (gas clouds, etc.)

- **Nonthermal sources:**
  - $T_b$ is related to the **mean particle energy** in the source (equivalent to $kT$)

- These statements are true for **incoherent sources** (particles radiating independently)

- **Coherent sources:**
  - Jupiter, solar bursts, brown dwarfs, interstellar and circumstellar masers, pulsars
  - $T_b$ is much larger than any possible physical temperature or mean particle energy ($10^{42}$ K for pulsars)

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**Radio Transient Phase Space**

Equivalent brightness temperature (RJ regime):

$$T_b = \frac{S}{2k\Delta\Omega}$$

Filling phase space with hypothetical new discoveries:

- Maximal giant pulse emission from pulsars
- Prompt Gamma-ray emission
- Evaporating black holes
- ETI’s asteroid radar
- What else?
Radio Telescopes as Radiometers: A radio telescope used as a single antenna with a radiometer measures power, not brightness or specific intensity. The power level for a radio telescope may be expressed as an integral

\[ P_\nu = \frac{1}{2} A_e \int d\Omega d\nu B_\nu(\Omega) P_\nu(\Omega), \]  

where

1. The factor of 1/2 accounts for the acceptance by a single antenna of only a single polarization whereas the radiation field from the source is assumed to be unpolarized (equal power in two opposite or orthogonal polarizations).
2. \( A_e \) = effective area of antenna \( \equiv \eta \times \text{geometric area} \approx 0.6\pi R^2 \), where \( \eta \) is the aperture efficiency of the telescope that measures the uniformity by which the main reflector is illuminated by the feed antenna at the focus.
3. \( B_\nu \) = brightness distribution of the sky as a function of frequency \( \nu \) and direction solid angle \( \Omega \).
4. \( P_\nu(\Omega) \) = the antenna power pattern. This is the response of the antenna as a function of direction, normalized so that the maximum response is 1 in the boresight direction and < 1 in other directions. The width of the ‘main lobe’ of the power pattern is \( \theta_K = \text{FWHM} \) (full width at half maximum) \( \approx \lambda/D_A \), \( D_A \) = antenna diameter. The solid angle of the main beam is \( \Omega_A = \int_{\text{main beam}} d\Omega P_\nu(\Omega) \approx \pi \theta_K^2/2 \).
5. \( \Delta\nu \) = receiver bandwidth = range of frequencies centered on some frequency \( \nu \). We always have \( \Delta\nu < \nu \).
6. Equation 3 can be viewed as a convolution integral of the antenna power pattern with the sky. In a drift scan of the sky, the measured width of the response will be a combination of the beam width and the source size.

Extended Sources: For a source with extent \( \Omega_s \gg \Omega_A \) (such as the 3K cosmic background), the brightness distribution is constant over the telescope beam and over the receiver bandpass, so

\[ P_\nu \approx \frac{1}{2} \Delta\nu A_e \Omega_A B_\nu. \]  

Recall that \( A_e \propto D_A^2 \) while \( \Omega_A \propto (\lambda/D_A)^2 \). Their product is therefore independent of the telescope diameter; moreover, it can be shown that \( A_e \Omega_A \propto \lambda^2 \). Using this fact and assuming that the extended source radiates as a blackbody, we get

\[ P_\nu \approx kT\Delta\nu. \]  

Thus (1) measurement of the power level of an extended source determines the radiation temperature of the source (the radiation temperature may or may not be related to the physical temperature in the source); (2) the ability to measure the temperature is just as good for a small antenna as a large one, to the extent that the source is extended for both. For example, extended 21 cm emission from the Galaxy is as easily detected with a 4 meter antenna as with a 300 meter antenna (Arecibo).
Point Sources: If $\Omega_s \ll \Omega_A$, the source is said to be a point source, since it is unresolvable with the telescope beam. Now, assuming the antenna power response $P_n$ to be constant over the source (and that the beam center points directly at the source $\rightarrow P_n = 1$), we have

$$P_\nu \approx \frac{1}{2} \Delta \nu \Delta A \Omega_s B_\nu(\max). \quad (6)$$

Flux density: Another useful quantity is flux density $F_\nu$ (flux = rate at which energy passes through a surface; ‘density’ refers to flux per unit frequency). Flux density has units of energy/time/area/frequency. A useful unit of flux density is the Jansky (named after Karl Jansky who, for Bell Labs in the early 1930’s, discovered radio emission from the Galactic center while investigating sources of noise in trans-Atlantic radio communications):

$$1 \text{ Jy} \equiv 10^{-26} \frac{\text{watts}}{\text{m}^2 \text{Hz}} \equiv 10^{-23} \frac{\text{erg}}{\text{s cm}^2 \text{Hz}} \quad (7)$$

The flux densities of thousands of radio sources are tabulated, having been determined in sky surveys made at frequencies from about 25 MHz up to 100 GHz (a range of 4000:1).

The relationship between flux density and brightness distribution is

$$F_\nu = \int d\Omega B_\nu(\Omega). \quad (8)$$

We can rewrite this expression as

$$F_\nu \equiv \Omega_s B_\nu(\max), \quad (9)$$

where we effectively define the solid angle of the source $\Omega_s$.

Returning to the telescope response for a point source and using the flux density, we have

$$P_n = \frac{1}{2} \Delta \nu A \Omega_s F_\nu. \quad (10)$$

Note that, unlike an extended source, the power measured from a point source scales linearly with the telescope area; bigger antennas measure more power.
Single polarization system, one pixel

\[ E(x, t) \]

\( \hat{p} = \text{unit vector for antenna polarization (} \hat{x}, \hat{y}, \text{RHCP, LHCP)} \)

voltage \( \propto \hat{E} \cdot \hat{p} \)

**Modern: digital vs earlier approach: analog**

LNA

Bandpass Filter

Square-Law Detector

Integrator \( \overline{I(t)} \)

\[ \overline{I(t)} \propto |v(t)|^2 \]

Stochastic signal

The time-integrated output has fluctuations given by the radiometer equation

\[ \frac{\sigma_t}{\langle I \rangle} = \frac{1}{\sqrt{\Delta f T}} \]

The radiometer equation is a consequence of summing an ensemble of sinusoids with random phases and the central limit theorem.

**Telescope Response in Terms of Effective Temperature:** Suppose we assume that the power level we measure is due to an extended source at temperature \( T \) whereas it actually is due to a point source of flux density \( F_\nu \). Equating equations (1) and (2) and solving for temperature gives

\[ T = \left( \frac{A_e}{2k} \right) F_\nu. \] (11)

The quantity \( A_e/2k \) serves as a conversion factor from flux density to temperature:

\[ G = \frac{A_e}{2k} = 2.7 \times 10^{-3} \text{ K/Jy} \quad \text{D} = 4 \text{ m} \]

\[ = 1.5 \times 10^{-1} \text{ K/Jy} \quad \text{D} = 300 \text{ m (Arecibo)} \] (12)

As an example, a 1 Jy source produces a 15K deflection at Arecibo compared with a system temperature that is 40K at 1.4 GHz or 100K at 0.4 GHz. This is a very large deflection.
**System Temperature and the Radiometer Equation:** It is convenient to describe the properties of the receiver system as well as the radiation field in terms of temperature. The system temperature measures the power level when the telescope is pointed at ‘blank’ sky and includes contributions from the sky background and noise in antenna, cable, and receiver components. The system temperature quantifies the mean power level. All contributions to the radiometer temperature are random in nature, so the receiver output is actually noiselike. This is easily demonstrated by viewing the signal on an oscilloscope or listening to the signal through a loudspeaker. (You can demonstrate this with your FM stereo by listening to the hiss when it is tuned in between two stations; some of this hiss is due to synchrotron radiation from cosmic ray electrons in our galaxy’s magnetic field).

Radiometer noise in a receiver behaves according to the familiar $1/\sqrt{N}$ law from statistics. That is, if we have a receiver with bandwidth $\Delta\nu$ and if we average the signal output over a time $\tau$, the number of independent fluctuations that is summed is $N = \Delta\nu\tau$. The root mean square fluctuation $\sigma_T$ of the radiometer divided by the mean temperature $T_{sys}$ is the radiometer equation:

$$\frac{\sigma_T}{T_{sys}} = \frac{1}{\sqrt{\Delta\nu\tau}}$$  \hspace{1cm} (13)

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**Demonstration of Radio Spectral Analysis**

- Radio spectrum computed from real-time FFT
- Bandpass defined by several elements in the receiver chain
- Noise = random ~ “white” noise = radiometer noise
- RFI = radio frequency interference
- Broad spectral line = Galactic atomic hydrogen
- Effective temperature of the spectral line ~ spin temperature of H ~ ambient temperature
- Needed for any telescope usage: how do we calculate noise and signal levels?
3.8 m Space Sciences Radio Telescope

3.8m diameter
f/0.4 prime focus optics
Single polarization channel
RF = 1.4 GHz
Three mixing stages
Can detect the 21 cm atomic hydrogen line as well as with the Green Bank Telescope (100m) and Arecibo (305m)
Why?

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Block diagram of “front-end” receiver

Figure 1: Diagram of three panels
Detectability of sources: The level of radiometer noise determines how well we can measure the system temperature. More importantly, it determines how well we can determine that, rather than looking at just blank sky, the telescope is pointed towards an actual source. In order to be detectable, the source must increase the system temperature by an amount that is distinguishable from the noise. A useful rule of thumb is that a source must increase the temperature by an amount that is 5 times the statistical variation; i.e.

\[ T_{\text{source}} \equiv G F_{\nu} \geq 5\sigma_{F} = \frac{5T_{\text{sys}}}{(\Delta\nu \tau)^{1/2}} \]  \hfill (14)

Equivalently, the flux density must satisfy:

\[ F_{\nu} \geq \frac{5T_{\text{sys}}}{G(\Delta\nu \tau)^{1/2}} \]  \hfill (15)

Clearly, if we maximize the product of bandwidth and time constant, we can measure weaker sources. For \( T_{\text{sys}} = 100 \text{K}, \Delta\nu = 40 \text{ MHz}, \text{and } \tau = 1 \text{ sec}, \) and using the above values for \( G, \) we find detection levels

\[ F_{\nu} \geq \frac{0.079 \text{ Jy}}{G} = \begin{array}{c} 30 \text{ Jy} \quad 4 \text{ m} \\ 0.0053 \text{ Jy} \quad 300 \text{ m}. \end{array} \]  \hfill (16)
The way sources are detected is by performing a differential measurement on and off the source (e.g. measure the radiometer level when the telescope beam is pointed toward the source; subtract from this the level when it is pointed at one or more adjacent positions) and testing whether the difference is larger than the noise fluctuations. Alternatively, sometimes drift scans are performed by pointing the telescope to a position through which the source will drift due to Earth’s rotation. A point source will produce a response that is proportional to a one-dimensional cut through the beam $P_\nu(\Omega)$. Drift scans are useful for 1) discovering new sources; 2) mapping out the beam; and 3) testing the pointing of the telescope.

Other factors: There is a limit to how well one can increase the time-bandwidth product as opposed to increasing $G \propto A$. This is because other factors come into play, especially source confusion, which results from there being multiple sources within the telescope beam at a given sky position. Moving from one sky position to another, these sources change, thereby producing a radiometer fluctuation that can exceed that produced by radiometer noise alone. If so, measurements are said to be ‘confusion limited’ rather than radiometer noise limited. Telescopes with larger beams (i.e. small antennas at large wavelengths) have larger confusion variations. The rms confusion fluctuation is roughly

$$\sigma_c \approx 3\sigma_0 \left( \frac{\nu}{1 \text{ GHz}} \right)^{-0.7} \Omega_d \text{ Jy.}$$  \hspace{1cm} (17)$$

For the Arecibo telescope at 1.4 GHz, $\sigma_c \approx 2\text{mJy}$. A source must be stronger than $5\sigma_c \approx 10 \text{ mJy}$ to be convincingly detected.
Single polarization system, one pixel

$\hat{E}(x, t) = \text{unit vector for antenna polarization } (\hat{x}, \hat{y}, \text{RHCP, LHCP})$

Voltage $\propto \hat{E} \cdot \hat{p}$

Modern: digital vs earlier approach: analog

Real and imaginary voltage (in phase and quadrature components)

Quadrature baseband mixer:

$$I_t = \cos 2\pi \nu_{LO} t$$

$$Q_t = \sin 2\pi \nu_{LO} t$$

Utility of Baseband Signal

- Complex baseband voltage is related to the Fourier transform of the selected electric field
- Can show that

$$E_\Delta(t) = \{ \varepsilon(t)e^{2\pi i\nu_{RF} t} \}$$

$E_\Delta(t) =$ selected (scalar) $E(t)$ field in RF bandwidth

$\varepsilon(t) =$ complex baseband field

- Significance?
  - Access to fundamental units of radiation in relativistic sources
  - Have access to phase as well as amplitude of the radiation field
  - Important for pulsar applications (coherent dedispersion)
  - Necessary for interferometry
1. Two examples of the power of phase

• Imaging with aperture synthesis = manipulation of the correlation function between fields measured between multiple pairs of antennas

• Dedispersion of a pulsar signal to remove the effects of propagation through the ionized interstellar medium (deconvolution of digital data)
Sub-nanosec shot pulse from the Crab Pulsar

Requires digital filtering of the electric field components selected over a 2.5 GHz bandwidth to remove phase wrapping imposed by dispersive propagation through the interstellar medium.