A6525 Lecture 9 = Radio Lecture 4

- Manipulating phase
- Measuring polarization:
  - Polarization states
  - Stokes parameters, Jones matrices, Mueller matrices
  - Cross coupling, calibration
- Propagation effects in plasmas
  - Dispersion
  - Faraday rotation
  - Scattering

Intensities, Luminosities, Transmitters, etc.

Specific intensity $I_\nu = \text{erg s}^{-1} \text{cm}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$

Figure 1: Power received in $dA \, dv \, d\Omega = I_\nu (dA \cos \theta) \, d\Omega$

Calculations of $I_\nu$ from source properties: consider a spherical, isotropically radiating source

Solid angle of source, $\Omega_s$. 
For uniform specific intensity across the apparent disk with diameter $\theta_s$:

$$I_{\nu} = \begin{cases} \text{constant} & \text{across disk} \\ 0 & \text{outside disk} \end{cases}$$

For a spherical coordinate system with the $z$ axis toward source center

$$\Omega_s = \frac{\int d\Omega I_\nu(\theta_s, \phi)}{I_{\nu_{\text{max}}}} = \int_d \int_0^{\theta_s/2} d\phi d\theta \sin \theta = 2\pi(1 - \cos \theta_s/2)$$

For small angles

$$\Omega_s \approx \pi(\theta_s/2)^2.$$

Isotropically radiating source of diameter $2R$:

$$\Omega_s \approx \pi(R/r)^2.$$
For beamed sources such as AGNs, \( \gamma \)-ray bursts, pulsars, other transients:

\[
4\pi \rightarrow \Omega_r = \text{solid angle of radiation beam}
\]

so

\[
I_\nu = \begin{cases} 
\frac{L_\nu}{\Omega_r r^2 \Omega_s} = \frac{L_\nu}{\pi \Omega_r R^2} & \text{if detector is in radiated beam} \\
0 & \text{if not.}
\end{cases}
\]

**Brightness temperature of a transmitting antenna**

Consider a transmitter emitting a luminosity \( L \equiv p_t \) (power):

- \( \Delta\nu_t = \text{bandwidth into which power is radiated (small e.g. } \lesssim 1 \text{ Hz)} \)

\( L_\nu = \frac{dL}{d\nu} = \frac{p_t}{\Delta\nu_t} = \text{Luminosity per unit frequency} \)

\[
I_\nu = \frac{L_\nu}{\pi \Omega_r R^2} = \frac{p_t/\Delta\nu_t}{\pi (\lambda/2R)^2 R^2} = \frac{4}{\pi} \cdot \frac{p_t}{\lambda^2 \Delta\nu_t}
\]

Note independent of transmitter size, \( R \).

- Figure 2: Transmitter emitting power \( p_t \)
- into a solid angle \( \Omega_r = (\pi/4) R^2 \)
**Brightness temperature:** Rayleigh-Jeans regime

\[ I_\nu = \frac{2kT_b}{\lambda^2} = \frac{4}{\pi} \cdot \frac{p_t}{\lambda^2 \Delta \nu_t} \]

\[ \implies T_b = \frac{2}{\pi} \cdot \frac{p_t}{k \Delta \nu_t} \]

\( T_b \) is independent of distance and wavelength!

Evaluate:

\[ T_b = \frac{2}{\pi} \cdot \frac{p_t}{1.38 \times 10^{-16} \text{ erg K}^{-1} \text{ Hz}^{-1}} \cdot \frac{\text{1 watt}}{1 \text{ Hz}} \cdot \frac{(10^7 \text{ erg s}^{-1}/\text{watt}) \cdot \text{10} \text{ watts}}{\Delta \nu_t (\text{Hz})} \]

\[ = 10^{22.7} \text{ K} \times \frac{p_t(\text{watts})}{\Delta \nu_t (\text{Hz})} \]

E.g. Arecibo planetary radar: \( p_t = 10^6 \text{ watts}, \Delta \nu_t = 1 \text{ Hz}: \]

\[ T_b = 10^{28.7} \text{ K} \]

Questions:
1. How far can the Arecibo radar be detected with an Arecibo-type telescope?
2. What is the brightness temperature of your cell phone?
Effects that change the specific intensity:

1. Emission: $dI_{\nu} = \epsilon_{\nu} ds$, $\epsilon =$ emissivity
2. Absorption: $dI_{\nu} = -\kappa_{\nu} I_{\nu} ds$, $\kappa_{\nu} =$ absorption coefficient
3. "Negative" absorption from stimulated emission (masers, lasers) ($\kappa_{\nu}u < 0$)
4. Scattering:
   (a) IR $\rightarrow$ X-rays: interstellar grains, atmospheric turbulence
   (b) radio: electron density fluctuations in the interstellar medium
5. Scintillations caused by scattering: constructive and destructive interference
6. Polarization conversion effects: polarized components of $I_{\nu}$ can intermix as they propagate (e.g. Faraday rotation and similar effects in AGNs, pulsar magnetospheres)

Other factors: There is a limit to how well one can increase the time-bandwidth product as opposed to increasing $G \propto A$. This is because other factors come into play, especially source confusion, which results from there being multiple sources within the telescope beam at a given sky position. Moving from one sky position to another, these sources change, thereby producing a radiometer fluctuation that can exceed that produced by radiometer noise alone. If so, measurements are said to be 'confusion limited' rather than radiometer noise limited. Telescopes with larger beams (i.e. small antennas at large wavelengths) have larger confusion variations. The rms confusion fluctuation is roughly

$$\sigma_c \approx 3700 \left( \frac{\nu}{1 \text{ GHz}} \right)^{-0.7} \Omega_d \text{ Jy}. \quad (17)$$

For the Arecibo telescope at 1.4 GHz, $\sigma_c \approx 2\mu\text{Jy}$. A source must be stronger than $5\sigma_c \approx 10 \text{ mJy}$ to be convincingly detected.

Figure 6. Confusion profile. This profile plot shows the 3 GHz confusion amplitude in an 8 arcsec FWHM beam, truncated at $\mu\text{mJy beam}^{-1}$. 
Radio source distributions

- Flux-density distribution for extragalactic sources
- “log N – log S”
- Normalized to Euclidean, standard-candle distribution:
  \[ dN/dS \sim S^{-5/2} \]

Figure 1. Published 1.4 GHz source counts. The brightness-weighted source count \( S^{-5/2} \) is proportional to the contribution per decade of flux density to the sky background temperature \( T_b \). The filled points at \( \log(S) \) are from Condon (1986a) and Mitchell & Condon (1985); the polygon encloses the range of 1.4 GHz counts consistent with confusion (Mitchell & Condon 1985), and the straight line inside the box is the best power-law fit to the confusion data. The open dots and their power-law fit (upper straight line) indicate the Owen & Morrison (2008) source count. The solid curve in the Condon (1986b) model count composed of sources powered primarily by AGNs (dashed curve) and by star formation (dotted curve). Left ordinate: \( \log(S) \) at 1.4 GHz. Right ordinate: \( \log(dN/dS) \) (unm) to the 1.4 GHz background per decade of flux density.

Classic Radio Interferometer Array

- The Karl G. Jansky Very Large Array in 1km D-configuration
  - Resolution of 1000-m aperture, area of 130m aperture

Field-of-view is diffraction limit of element apertures

- VLA configuration D (1km at 30cm = 69')
- VLA configuration A (36km at 30cm = 2')

\( \lambda/D \sim 10^\prime \) at 1GHz (30cm)
Future Radio Interferometer Array

- Science observing since March 2010 – a Laboratory on the Sky!
- Future possibilities – the road towards the Square Kilometre Array and the LSST next decade – the Jansky VLA is a SKA Science Proving Ground!

The VLA Sky Survey (VLASS) Initiative

- Announced 11 July 2013: Community-led Program to define a new radio sky survey using the upgraded Karl G. Jansky VLA
  – Previous centimeter-wave VLA Surveys: NVSS & FIRST 1993-2002
  – Open *international* participation, public data and products
  – VLASS data public from start (no proprietary period)
- Fall 2013: Issued a call for White Papers - 21 Papers!
- AAS workshop 5 January 2014 (~50 attendees, see online)
- 2014: Survey Science Group (SSG), working groups formed
  – survey proposal developed, drafts posted, comments, refined
  – technical implementation plan (TIP: Myers et al.)
- Jan 2015: Final Proposal posted ALL-SKY + DEEP
  – ~9000 hrs. over 7 years (6 config. cycles, A+B config.)
  https://science.nrao.edu/science/surveys/vlass
Key Science Cases – Highlights

• Medium/Deep Fields for Galaxy Evolution & Cosmology
  – AGN and Clusters of Galaxies, Feedback
  – Star-forming Galaxies
  – Weak Lensing

• Large Area Survey for Transients & Faraday Tomography
  – Full Polarimetry for B-field Studies
  – EM Counterparts to GW events (LIGO/VIRGO)
  – Radio Bursts on timescales from 1ms to >1 year

• Galactic Plane and Center
  – Atomic and Molecular Lines from 0.2-50 GHz
  – Stars and Stellar Systems

**Notional schedule (as of July 2015)**

<table>
<thead>
<tr>
<th>Date</th>
<th>Activity</th>
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<tbody>
<tr>
<td>2015 March 4 – 6</td>
<td>External Community Review (Socorro)</td>
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<tr>
<td>2015 March – 2015 Aug</td>
<td>Set up Project Office, draft workplan, allocate resources</td>
</tr>
<tr>
<td>2015 March – 2016 May</td>
<td>Test &amp; Development Program carried out</td>
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<tr>
<td>2015 November</td>
<td>VLASS Preliminary Design Review (PDR)</td>
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<tr>
<td>2016 May 27</td>
<td>Start of 2016A B-config (VLASS pilot observations possible)</td>
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<tr>
<td>2016 June</td>
<td>VLASS Critical Design Review (CDR), final go/no-go</td>
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<tr>
<td>2016 Aug 29</td>
<td>End of 2016A B-config (nominal, without VLASS)</td>
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<tr>
<td>2016 Oct 3</td>
<td>End of 2016A B-config (with a 1 month extension for VLASS)</td>
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<tr>
<td>2017 Apr 3</td>
<td>Delivery of B-config Epoch 1 (6 months: ALL-SKY I only)</td>
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<tr>
<td>2017 Oct 3</td>
<td>Delivery of B-config Epoch 1 (12 months: Pol.)</td>
</tr>
<tr>
<td>2017 Sep</td>
<td>VLASS Cycle 2 observations commence (B-config)</td>
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Headline Science: Faraday Tomography

- NVSS+: Taylor, Stil, Sundstrom 2009
  - $3 \times 10^4$ sources, ~1/deg$^2$
- VLASS:
  - conservative estimate
  - $2 \times 10^5$ sources, ~6/deg$^2$
- Science:
  - map our Galactic B
  - B through the cosmic web
  - evolution with $z$

Single polarization system, baseband signal

$\mathbf{E}(x, t)$

$\hat{p}$ = unit vector for antenna polarization ($\hat{x}$, $\hat{y}$, RHCP, LHCP)

time-varying voltage $\propto \dot{\mathbf{E}} \cdot \hat{p}$

Modern: digital vs earlier approach: analog

Quadrature baseband mixer:

In phase component

Quadrature component
Utility of Baseband Signal

- Complex baseband voltage is related to the Fourier transform of the selected electric field
- Can show that
  \[ E_{\Delta}(t) = \{\varepsilon(t)e^{2\pi i\nu_{RF}t}\} \]
  \( E_{\Delta}(t) = \) selected (scalar) \( E(t) \) field in RF bandwidth
  \( \varepsilon(t) = \) complex baseband field

- Significance?
  - Access to fundamental units of radiation in relativistic sources
  - Have access to phase as well as amplitude of the radiation field
    - Important for pulsar applications (coherent dedispersion)
    - Necessary for interferometry
Why are complex signals useful in the analysis of receiver systems?

1. Complex quantities like $e^{i\omega t}$ are easier to manipulate than $\cos \omega t$, etc.

2. Many receiver systems are narrowband i.e. $\Delta \nu \ll \nu$, for which the signal may be written as complex components.

   e.g. 
   
   \[ v(t) \propto E_\Delta(t) = R \{ e^{i\omega_0 t} \} \]

   where
   
   $v(t) = \text{real voltage}$
   $E_\Delta(t) = \text{narrowband electric field (scalar here)}$
   $e^{i\omega_0 t} = \text{fastly varying “carrier” frequency}$

3. The output of a quadrature baseband mixer yields I,Q components that may be considered the real and imaginary parts of $e(t)$.

Radiation Field and Receiver Voltages

A basis vector for EM waves is a plane wave

\[ e^{i(k \cdot x - \omega t)} \]

so a general radiation field has a Fourier representation

\[ E(x, t) = \int d\mathbf{k} d\omega \hat{E}(\mathbf{k}, \omega) e^{i(k \cdot x - \omega t)} \cdot \mathbf{p} \]

An observer at $\mathbf{x}_{\text{observer}}$ using a telescope that is selective in direction, polarization, and frequency via the antenna response and receiver filters, etc. can measure a voltage at RF frequencies

\[ V_{\text{RF}}(t) \propto \int d\mathbf{k} d\omega \hat{E}(\mathbf{k}, \omega) \cdot \mathbf{p} e^{i(k \cdot x - \omega t)} \]

$\mathbf{p}$ = antenna polarization vector, assumed independent of direction and frequency (not generally true)

By inspection, the RF voltage contains the information in the selected Fourier components. Through appropriate manipulation, these Fourier components can be used.
For a narrowband system with center frequency $\nu_0$ and bandwidth $\Delta \nu$, the RF voltage fluctuates on time scales $\sim \nu_0^{-1}$. At GHz frequencies this is $\sim 1$ ns.

While digitizers (AGCs) exist that can sample this fast, much higher frequencies cannot be sampled. For this and other reasons, it is useful to mix the signal to lower frequencies. The ultimate shift is all the way to baseband, where $\nu_0$ is mapped to zero frequency and the bandwidth is $\Delta \nu / 2$.

Puzzle: does this mean that the bandwidth has been halved and that we can satisfy the sampling theorem differently than before?

A: No, the same number of samples is needed because the sampling must be of a complex signal (real and imaginary parts).

What is the response of the system to a point source that emits an impulse (delta functions in direction and time)?

A: From the student!

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**Two examples of the power of phase**

- Imaging with aperture synthesis = manipulation of the correlation function between fields measured between multiple pairs of antennas
- Dedispersion of a pulsar signal to remove the effects of propagation through the ionized interstellar medium (deconvolution of digital data)
Cygnus A

Radio image

Very Large Array

Aperture Synthesis

Sub-nanosec shot pulse from the Crab Pulsar

Crab Pulsar

8 – 10.5 GHz

Requires digital filtering of the electric field components selected over a 2.5 GHz bandwidth to remove phase wrapping imposed by dispersive propagation through the interstellar medium.
Propagation through the interstellar plasma

Refractive indices for cold, magnetized plasma

\[ n_{e,r} \sim 1 - \frac{\nu_p^2}{2\nu^2} + \frac{\nu_B^2}{2\nu^3} \]
\[ \nu \gg \nu_p \sim 2 \text{kHz} \quad \nu \gg \nu_B \sim 3 \text{Hz} \]

Propagation velocities are frequency dependent:

Phase velocity: \( v_p = \frac{\omega}{k} = \frac{c}{n_e} \)

Group velocity: \( v_g = \frac{\partial \omega}{\partial k} = \frac{\partial}{\partial k} \left( \frac{kc}{n_e} \right) \)

Group delay = \( \Delta (\text{Time of Arrival}) \)

\[
\begin{align*}
t &= t_{\text{DM}} \pm t_{\text{RM}} \\
t_{\text{DM}} &= 4.15 \text{ ms } DM \nu^{-2} \\
t_{\text{RM}} &= 0.18 \text{ ns } RM \nu^{-3}
\end{align*}
\]

Dispersion Measure \( DM = \int ds n_e \) units: pc cm\(^{-3}\)
Rotation Measure \( RM = 0.81 \int ds n_e B_{\parallel} \) units: rad m\(^{-2}\)

Interstellar Dispersion

- Ionized gas in the ISM is dispersive: refractive index is frequency dependent (chromatic)
  - The group velocity of a pulse is therefore frequency dependent
  - The arrival time of a pulse is chromatic
  - The phases of different Fourier components are altered systematically

- The spread in arrival times depends on the receiver bandwidth

\[
\Delta t = \frac{8.3 \mu s \text{ DM } \Delta \nu}{\nu^3}
\]
\[ \nu = \text{ frequency in GHz} \]
\[ \Delta \nu = \text{ bandwidth in MHz} \]
\[ DM = \int ds n_e (s) = \text{dispersion measure pc cm}^{-1} \]
\[ n_e = \text{electron density} \]

**Crab pulsar:**

\( DM = 56.791 \text{ pc cm}^{-3} \)
\( \nu = 9 \text{ GHz} \)
\( \Delta \nu = 2.5 \text{ GHz} \)
\( \Delta t = 1620 \text{ \mu s} \)
A Single Dispersed Pulse from the Crab Pulsar

S ~ 160 x Crab Nebula
~ 200 kJy
Detectable to ~ 1.5 Mpc
with Arecibo

\[ t_{\text{DM}} = 4.15 \text{ ms \, DM \, } \nu^{-2} \]
\[ \nu \text{ in GHz} \]
\[ n_e \text{ in cm}^{-3} \]

Dispersion measure:
\[ \text{DM} = \int ds n_e(s) \text{ pc cm}^{-3} \]

Coherent Dedispersion:
Bandwidth vs. Date

Doubling time: 1.2 years
Polarization & Polarimetry

- Polarization of EM radiation carries extra information:
  - Kinematic information about emitting particles
    - E.g. magnetic field directions
    - Synchrotron radiation, spinning stars, spinning grains
  - Symmetries/selecton rules in quantum states (e.g. masers)
  - Reflection and scattering
- A monochromatic wave is 100% polarized and it takes 3 parameters to describe its polarization state
- A signal with finite bandwidth can be partially polarized, so a fourth parameter is needed to describe that partial state.
- It is far better to work with intensity-like quantities than field-like quantities, hence the utility of Stokes parameters.
- Propagation, radiative transfer, and instrumental polarization can be described with a 4 x 4 Mueller matrix that modifies the Stokes 4-vector
  - Or equivalently with Jones matrices (2 x 2) that modify the Jones vector

TENTATIVE DETECTION OF ELECTRIC DIPOLE EMISSION FROM RAPIDLY ROTATING DUST GRAINS

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ABSTRACT

We present the first tentative detection of spinning dust emission from specific astronomical sources. All other detections in the current literature are statistical. The Green Bank 140 foot telescope was used to observe 10 dust clouds at 5, 8, and 10 GHz. In some cases, the observed emission was consistent with the negative spectral slope expected for free-free emission (thermal bremsstrahlung), but in two cases it was not. One H II region (LPH 201.663 + 1.643) yields a rising spectrum, inconsistent with free-free or synchrotron emission at the ~10 σ level. One dark cloud (L1622) has a similar spectrum with lower significance. Both spectra are consistent with electric dipole emission from rapidly rotating dust grains (“spinning dust”), as predicted by Draine & Lazarian.

Subject headings: cosmic microwave background — diffuse radiation — dust, extinction — ISM: clouds — radiation mechanisms: thermal — radio continuum: ISM
Wave Propagation Effects

\[ E(\omega) \]

media
- IGH, ISM, IOP
- atmosphere, troposphere
- as a index of refraction

\[ \text{modified } E(\epsilon, \omega) \]

Effects:
- refraction angle of arrival vs \( \lambda \)
- dispersion group velocity, time of arrival vs \( \lambda \)
- scattering fluctuations in \( \eta \)
- Faraday rotation polarization effects due to birefringence

As probes of intervening media
As effects that degrade astronomical measurements & surveys

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Random Light

Fluctuation properties of optical light \( F_0(E) \)

\[ \text{Re}(E) \]

\[ \text{Im}(E) \]

Fig. 1.6 (a) Probability distribution for the amplitude and phase of the electric field at \( \omega = \omega_0 \). a) A lognormal distribution in the form \( \text{Re}(E) \) and \( \text{Im}(E) \), with the distribution of 

\[ f(E) \] in the form, which can be Lorentzian, Gaussian, or an intermediate function, and is related to the probability for the collision-bending model assumed here. The probability distribution \( f(E) \) can be expanded into a probability distribution \( f(E) \) with the help of eqs. (3.13).

Problem 2.2: Prove that the probability for an instantaneous measurement of the peak-valued intensity to fall a value between \( E \) and \( E + \delta E \) is

\[ f(E) \delta E = \frac{1}{\pi} \delta E \delta \left( E - E_0 \right) . \]
Lasers are used to measure and control the phase of the output beam. The diagram illustrates the phase distribution of the laser light, with arrows indicating the direction of the phase change. The phase changes are measured using interferometric techniques, which allow for precise control of the laser output. The phase changes are critical for applications such as optical communications and optical metrology.

The polarized signal is shown in the diagram, with arrows indicating the direction of polarization. The unpolared signal is also shown, with arrows indicating the direction of the electric field. The phase changes are measured using interferometric techniques, which allow for precise control of the laser output. The phase changes are critical for applications such as optical communications and optical metrology.
Polarization and Polarimetry

Consider a monochromatic wave propagating along the $z$ axis:

$$E(z, t) = \begin{bmatrix} E_1 \sin(\omega t - kz) \\ E_2 \sin(\omega t - kz + \phi) \end{bmatrix}$$

Implicit: the amplitudes $E_{1,2}$ and phase $\phi$ are time and $z$ independent. $E_{1,2}$ and $\phi$ characterize the net polarization state.

The plane containing $\hat{z}$ and $E$ is often called the plane of polarization.

The locus of points described by $E(z, t)$ is an ellipse whose size, shape, and orientation are constant unless there is absorption or Faraday rotation of the plane of polarization.

Consider a fixed point $z = 0$ and solve for a relation between the $E_x$ and $E_y$ components.

It can be shown that

$$AE_x^2 + BE_x E_y + CE_y^2 = 1$$

where

$$A = \frac{1}{(E_1 \sin \phi)^2}, \quad B = \frac{2 \cos \phi}{E_1 E_2 \sin^2 \phi}, \quad C = \frac{1}{(E_2 \sin \phi)^2}$$

The polarization ellipse is thus...
Points:

• Monochromatic wave: three numbers are needed to characterize the polarization state: $E_1, E_2, \phi$.
• At typical frequencies, the wave oscillates to rapidly to measure the polarization directly.
• Stokes parameters are intensity-like quantities that allow time-averaging to build up S/N.
• Noise-like signals can be 100% polarized but usually are only partially polarized.
• Four Stokes parameters are needed to characterize the polarization state of noise-like signals:

$$S = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} \quad Q, U \rightarrow \text{linear polarization} \quad V \rightarrow \text{circular polarization}$$

• The general state of polarization is

unpolarized wave field + 100% elliptically polarized wakefield

= partially polarized wakefield

• $I^2 \geq Q^2 + U^2 + V^2$
• Two radiation fields with the same Stokes parameters are equivalent.
• Unpolarized radiation can be formed by combining polarized wave fields with opposite polarizations with equal intensities

$$\implies S = \begin{pmatrix} I \\ 0 \\ 0 \\ 0 \end{pmatrix}$$
RADIO ASTRONOMICAL POLARIMETRY AND THE LORENTZ GROUP

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ABSTRACT
In radio astronomy, the polarimetric properties of radiation are often modified during propagation and reception. Effects such as Faraday rotation, receiver cross talk, and differential amplification act to change the state of polarized radiation. A general description of such transformations is useful for the investigation of these effects and for the interpretation and calibration of polarimetric observations. Such a description is provided by the Lorentz group, which is intimately related to the transformation properties of polarized radiation. In this paper, the transformations that commonly arise in radio astronomy are analyzed in the context of this group. This analysis is then used to construct a model for the propagation and reception of radio waves. The implications of this model for radio astronomical polarimetry are discussed.

Subject headings: instrumentation; polarimeters — polarization — techniques; polarimetric

41

Instrumental Polarization

Signal Model
Let $\hat{x}$ and $\hat{y}$ denote unit vectors in the two directions of linear polarization selected by the feed + OMT combination. We assume that the linear signals from the OMT are perfect. In practice, they are nonorthogonal and there is cross coupling; we consider cross coupling later. Here, we are concerned with conversion to circular. Total gains from primary through gregorian optics, feed, OMT, initial LNA, and paths in a quadrature hybrid are $g_{x,y}$. Here and in the following, all such gains are recognized to be functions of frequency, $\omega$. The hybrid combines the $x,y$ signals into effective unit vectors for nominal (primed) circular polarizations:

$$\hat{\epsilon}^\prime_{R,L} = \frac{1}{\sqrt{2}} \left[ g_x(\omega) \hat{x} \pm g_y(\omega) e^{i\phi(\omega)} \hat{y} \right],$$

(1)
$$\hat{\epsilon}_{R,L} = \frac{1}{\sqrt{2}} \left[ g_x(\omega)\hat{x} \pm ig_y(\omega)e^{i\delta\phi(\omega)}\hat{y} \right].$$

(2)

Given the unit vectors, the nominal right-and-left-hand circular polarizations may be written in terms of the (filtered) electric field \( \hat{E} \) (using \( ** \) to denote complex conjugate)

$$E'_{R,L} = \hat{E} \cdot \hat{\epsilon}_{R,L}^*.$$  

(3)

Measured voltages at the hybrid output \( V'_{R,L} = g_{R,L}E'_{R,L} \) involve gains \( g_{R,L} \) whose determination we assume is perfect.

\[ \ldots \]

An elliptically polarized signal may be written generally in terms of perfect circular unit vectors \( \hat{\epsilon}_{R,L} \)

$$\hat{E} \equiv E_R \hat{\epsilon}_R + E_L \hat{\epsilon}_L.$$  

(4)

The relationship between \( E' \) and \( E \) can be written in matrix form using a 2x2 Jones matrix to describe gain errors.

Equivalently, the nominal Stokes vector \( S' \) can be related to the true Stokes vector \( S \) using the Mueller matrix

$$S' = M S$$

When there are gain errors and phase errors the estimated Stokes parameters (primed) are related to the true Stokes parameters as

\[
\begin{pmatrix}
I' \\
Q'
\end{pmatrix} =
\begin{pmatrix}
\overline{G} & \frac{\Delta G}{2} & 0 & 0 \\
\frac{\Delta G}{2} & \overline{G} & 0 & 0
\end{pmatrix}
\begin{pmatrix}
I \\
Q
\end{pmatrix},
\]

\[
\begin{pmatrix}
U' \\
V'
\end{pmatrix} =
\begin{pmatrix}
0 & 0 & \gamma \cos \phi_\gamma & -\gamma \sin \phi_\gamma \\
0 & 0 & \gamma \sin \phi_\gamma & +\gamma \cos \phi_\gamma
\end{pmatrix}
\begin{pmatrix}
I \\
Q \\
U \\
V
\end{pmatrix},
\]

\[
\overline{G} = \frac{1}{2}(G_x + G_y)
\]

\[ G_{x,y} = |g_{x,y}|^2 \]

\[ \Delta G = G_x - G_y \]

\[ \gamma = (G_x G_y)^{1/2} \]

\[ \phi_\gamma = \phi_x - \phi_y - \delta \phi, \]

\[ \phi_{x,y} = \text{phases of complex gains} \]

\[ \delta \phi = \text{phase error in converting } x,y \text{ to RHCP and LHCP} \]
Cross Coupling

The analysis above assumes the $x$ and $y$ signals to be perfectly linearly polarized. In practice, these signals will be cross coupled according to a unitary matrix relating perfect (unprimed) to imperfect (primed) signals:

$$\begin{pmatrix} E'_x \\ E'_y \end{pmatrix} = \begin{pmatrix} \sqrt{1-\epsilon} & -\epsilon e^{i\psi} \\ +\epsilon e^{-i\psi} & \sqrt{1-\epsilon} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix},$$

(14)

where $\epsilon$ and $\psi$ are the amplitude and phase of the cross coupling. The transformation

$$\begin{pmatrix} \mathcal{U} \\ \mathcal{D} \end{pmatrix} = \begin{pmatrix} \mathcal{G}_x(1-2\epsilon) & -\Delta \mathcal{G}_\eta \cos \psi & -2\mathcal{G}_\eta \sin \psi \\ \mathcal{G}_y(1-2\epsilon) & \mathcal{G}_\eta \cos \psi & -\Delta \mathcal{G}_\eta \sin \psi \\ 2\gamma \mathcal{G} \sin \psi \cos \phi_\gamma & 2\gamma \mathcal{G} \cos \psi \cos \phi_\gamma & \mathcal{G} \cos \phi_\gamma (1-2\epsilon) - \gamma \sin \phi_\gamma \\ -2\gamma \mathcal{G} \sin \psi \cos \phi_\gamma & 2\gamma \mathcal{G} \cos \psi \sin \phi_\gamma & \mathcal{G} \sin \phi_\gamma (1-2\epsilon) + \gamma \cos \phi_\gamma \end{pmatrix},$$

(15)

where $\eta = \epsilon(1-\epsilon)^{1/2}$. With no cross coupling ($\epsilon = 0$), the matrix reduces to eqn (5).

---

Equatorial vs. Az-El Telescope Mounts

- **Equatorial:** the focal plane rotates at the same rate as the sky, so the $x,y$ polarizations maintain the same orientation with respect to the sky
  - Beneficial in many respects (e.g. for long integrations while tracking a source

- **Azimuth-elevation:** the focal rotates differentially with respect to the sky, so the projection of the $x,y$ polarizations varies as a source is tracked
  - Rotation angle = “parallactic” angle
  - Varies over a 180 deg range
  - Varies more rapidly the closer the source is to the zenith
  - Beneficial for polarization calibration
APPENDIX

1. Dualreck reception

Consider an electromagnetic wave incident on a radio observatory. The left and right circularly polarized components, \( E_L \) and \( E_R \) (complex), will be transformed by the following into linearly polarized components, \( E_{\parallel} \) and \( E_{\perp} \):

\[
\begin{bmatrix}
E_{\parallel} \\
E_{\perp}
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
E_L \\
E_R
\end{bmatrix}
\tag{A6}
\]

where \( \theta = \phi - \beta \) and \( \phi \) is a complete spectrum, implying that the radiation pattern is that of a two-panoramic antenna and signals are circularly polarized through spherical waves, following general theory and notation from Stinebring et al. 1984

\[
\begin{bmatrix}
E_{\parallel} \\
E_{\perp}
\end{bmatrix} =
\begin{bmatrix}
\cos \phi & \sin \phi \\
-\sin \phi & \cos \phi
\end{bmatrix}
\begin{bmatrix}
E_L \\
E_R
\end{bmatrix}
\tag{A7}
\]

where the real constants \( a_{\parallel}, a_{\perp}, b_{\parallel}, b_{\perp} \) specify the linearized response of the array. The overall phase shift, \( \phi \), between \( E_{\parallel} \) and \( E_{\perp} \) serves only to include the ground-based optical path for \( E_{\parallel} \) and \( E_{\perp} \), and will be accounted for in the subsequent calibration procedure.

A calibration procedure must also be used to account for the effect of high circularly polarized components in each antenna's response to the quasipolarized wave. We assume a source located at \( \phi = (345 \pm 5)^\circ \) and angle of the major axis, \( \chi = 125 \pm 10 \). Thus, even if the response components are elliptically polarized, they can be orthogonalized to one another.

\[
\begin{bmatrix}
a_{\parallel} \\
a_{\perp}
\end{bmatrix} =
\begin{bmatrix}
\cos \chi & \sin \chi \\
-\sin \chi & \cos \chi
\end{bmatrix}
\begin{bmatrix}
a_{\parallel} \\
a_{\perp}
\end{bmatrix}
\tag{A8}
\]

and the antenna will respond only to a generally polarized wave. In the most general case, when the response components are not orthogonally polarized, the antenna will be "blind" to radiation of a particular polarization (for the case of uncorrelated signal the power output of the blind will be many times larger than that of the two signals between the depolarized)

Using the definition of the dual-recall transform:

\[
\begin{bmatrix}
I \\
Q
\end{bmatrix} =
\begin{bmatrix}
\cos \phi & \sin \phi \\
-\sin \phi & \cos \phi
\end{bmatrix}
\begin{bmatrix}
E_L \\
E_R
\end{bmatrix}
\tag{A9}
\]

we have the transformation:

\[
\begin{bmatrix}
Q \\
I
\end{bmatrix} =
\begin{bmatrix}
\cos \phi & \sin \phi \\
-\sin \phi & \cos \phi
\end{bmatrix}
\begin{bmatrix}
\sum_{m} i_m E_{\parallel, m} \\
\sum_{m} i_m E_{\perp, m}
\end{bmatrix}
\tag{A10}
\]

A determination of the measuring parameters in equation (A9) would allow inversion of the transformation matrix and a solution for the true signals, \( E_L \) and \( E_R \), in terms of the measured \( I \) and \( Q \). Such a determination is only practical if a source of known characteristics and a single measurement are available. In the lack of a known source, there is no way to determine the receiving antenna response if more than a single measurement is not available. We assume that the receiving antenna response will be obtained from the calibration procedure.

A linear polarization of the source is:

\[
\begin{bmatrix}
\frac{1}{2} \Omega \\
\frac{1}{2} Q
\end{bmatrix}
\tag{A11}
\]

then we will have:

\[
\begin{bmatrix}
I = \frac{1}{2} \Omega \\
Q = \frac{1}{2} Q
\end{bmatrix}
\tag{A12}
\]

where \( \Omega \) is the polarization angle of the source between a linear wave front and one passing through the source and passing toward the elliptic plane. The linearized version of \( I \) and \( Q \) can be used to further the polarization response of the antenna. However, since we have determined only the magnitude of the response with no knowledge of the phase shift, we can only estimate the source power through the antenna and not the source polarization. To determine the source polarization, more than one measurement is needed since the antenna response is different and always different each time.
PULSAR POLARIZATION FLUCTUATIONS

The amplitude of the crosscoupling curve was well determined: \( \delta = 10.4 \pm 0.5 \)%. The phase of the crosscoupling curve varied somewhat from day-to-day, over approximately a 20° interval, most likely indicating differences in the position angle origin produced by different path lengths in the setup of the receiver. Plots of the residuals showed no obvious systematic trends, indicating that the assumption of negligible nonrotationality was well justified.

The application of the inverse transformation matrix was straightforward, since the use of measured position angle as the independent variable left no ambiguity about the phase origin of the correction matrix. Applying the alignment correction determined in Figure 39 to the waveforms plotted in Figure 38 results in the waveforms shown in Figure 40. The uniformity of the correction across the waveform suggests that there are no systematic errors in its determination.

The correction matrix, determined as above, was applied to all data obtained with the same observing setup. All data reported here have been corrected in this fashion. Limited attempts to determine correction coefficients from observations of other pulsars were only partially successful, primarily because a limited range of position angle; poorer quality waveforms, or both, limited the accuracy of the fits. Model parameters determined from observations of PSR 0823+26 and PSR 2020+28 were consistent with the amplitude and phase determined by modeling PSR 1929+10.
How to generate principal angles, C.f. reciprocating plane.

\[ \mathbf{c} \times \mathbf{a} = \mathbf{b} \]

Invariance

\[ \mathbf{b} = \mathbf{c} \times \mathbf{a} \]

Gives a non-singular transformation, provided it is well defined.

Example:

\[ \mathbf{a} \times \mathbf{b} = \mathbf{c} \]

How to get \( \mathbf{a} \) ?

To get \( \mathbf{a} \) in general.

\[ \mathbf{a} = \mathbf{b} \times \mathbf{c} \]

New angle

See Part III for particular cases.

The double summing

\[ \sum \frac{1}{1 + \lambda} \]

Half independent angle

\[ \frac{1}{2} \times \text{sin} \alpha = \frac{1}{2} \alpha \]

N.B. \( \beta < \alpha \) in the first quadrant.

\[ \beta = \text{arctan} \left( \frac{a}{b} \right) \]

algebraic:

\[ a = c \times d \] with \( c, d \) complex

\[ V(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} V(x) e^{-i\omega x} dx \]

\[ V(x) = \text{Fourier transform} \]

\[ V(\omega) = |V(\omega)| e^{i\phi(\omega)} \]

where phase \( \phi(\omega) \) independent of \( x(t) \).
To characterize the rate of purification of the
membrane, one requires
\[E, E', \text{ and } E''\] (3 reactions).

State variables: membrane states \[E, E', E''\]

Consider the following example:

\[\langle I \rangle = \langle I_1 \rangle + \langle I_2 \rangle + \langle I_3 \rangle\]

where \[\langle I \rangle = \text{mode average} = \frac{1}{N} \sum_{i=1}^{N} \langle I_i \rangle, 0 \leq \langle I \rangle \leq \text{max.}\]

\[\langle I_1 \rangle = (t_1^n \langle v_1 \rangle + t_2^n \langle v_2 \rangle) / (t_1 + t_2)\]

\[\langle I_2 \rangle = \langle v_1 \rangle / (t_1 + t_2)\]

\[\langle I_3 \rangle = \langle v_2 \rangle / (t_1 + t_2)\]

Note:

\[\langle Q \rangle = \langle E_2' \rangle - \langle E_3' \rangle = \langle (E_2 - E_3) \rangle\]

\[\langle W \rangle = 2 \langle E_1' (t_1 E_1) \rangle + 2 \langle E_2' (t_2 E_2) \rangle + 2 \langle E_3' (t_3 E_3) \rangle = 2 \langle E_1 \rangle \langle E_1 \rangle = \text{other}\]

\[\langle W \rangle = 2 \langle E_1' (t_1 E_1) \rangle + 2 \langle E_2' (t_2 E_2) \rangle + 2 \langle E_3' (t_3 E_3) \rangle = 2 \langle E_1 \rangle \langle E_1 \rangle = \text{other}\]
\[ \begin{align*}
E_1 \cdot E_2 & = 1 \\
\begin{bmatrix}
\rho_1 \\
\psi_1
\end{bmatrix} & \begin{bmatrix}
\rho_2 \\
\psi_2
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\end{align*} \]

\[ E_1 \cdot E_2 = 0 \Rightarrow \begin{bmatrix}
\rho_1 \\
\psi_1
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]

\[ E_1 \cdot E_2 = 0 \Rightarrow \begin{bmatrix}
\rho_1 \\
\psi_1
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]

\[ \text{The } \psi \text{ and } \phi \text{ are both specific polarizations. } \]

\[ \psi \text{ and } \phi \text{ are both specific polarizations. } \]

\[ \text{Consequence: } \forall \gamma > 0 \Rightarrow \text{Reff} * \text{orientation of } \frac{1}{\gamma} \]

\[ \psi \text{ is purely acute } \Rightarrow \begin{bmatrix}
\rho_1 \\
\psi_1
\end{bmatrix} \text{ is defined only for } \gamma = \text{ even} \]

\[ \text{in } \mathbb{R}^2 : \]

\[ \begin{bmatrix}
\rho \\
\phi
\end{bmatrix} \text{ in } \mathbb{R}^2 \text{ changes for different } \theta \text{.}
\]

\[ \text{Simple scale } \rightarrow \text{ can change } \begin{bmatrix}
\rho \\
\phi
\end{bmatrix} \text{ for } \theta
\]

\[ \text{Oval representation: } \]

\[ \text{The elliptical angle } \theta \text{ is important for visualization.}
\]

\[ \begin{bmatrix}
\rho \\
\phi
\end{bmatrix} \text{ in } \mathbb{R}^2 \text{ changes for different } \theta \text{.}
\]

\[ \text{Simple scale } \rightarrow \text{ can change } \begin{bmatrix}
\rho \\
\phi
\end{bmatrix} \text{ for } \theta
\]

\[ \text{Note particularly angle for measurement.}
\]

\[ \begin{bmatrix}
\rho \\
\phi
\end{bmatrix} \text{ in } \mathbb{R}^2 \text{ changes for different } \theta \text{.}
\]

\[ \text{Simple scale } \rightarrow \text{ can change } \begin{bmatrix}
\rho \\
\phi
\end{bmatrix} \text{ for } \theta
\]

\[ \text{Note particularly angle for measurement.}
\]

\[ \text{If } \theta \text{ changes on } \rho \text{, then } \theta \text{ is constant.}
\]

\[ \forall \gamma > 0 \Rightarrow \text{Reff} = \text{Reff} \text{ at } \theta \text{, constant}
\]

\[ \begin{align*}
\text{Subject to the condition of many physics (Units here):}
\end{align*} \]

\[ \text{and } \theta \text{ is returned from the point maximum toward the } \theta
\]
Some parameters in terms of circular electric field components:

With the unit x,y as basis vectors for $E$
we can decompose $E$ into $E_x = \text{right hand}$
and $E_y = \text{left hand}$.
These are basis vectors that are linearly related to the x,y basis components.

New

$$
\begin{align*}
\langle I \rangle &= \langle E_x^2 \rangle + \langle E_y^2 \rangle \\
\langle V \rangle &= \langle E_x^2 \rangle - \langle E_y^2 \rangle \\
\langle \varphi \rangle &= 2 \langle E_x(t) E_y(t) \rangle \\
\langle U \rangle &= 2 \langle E_x(t) E_y(t - t_0) \rangle
\end{align*}
$$

Rule of thumb:

If same in polarization simply multiply the right hand term with similarly polarized fields.
All else remain

Why: Can’t calculate products with time dependence when using quanta.

Two ways for Stokes parameters:

$$
\begin{pmatrix}
I \\
Q \\
U \\
V
\end{pmatrix} =
\begin{pmatrix}
\langle E_x^2 + E_y^2 \rangle \\
\langle E_x^2 - E_y^2 \rangle \\
2 \langle E_x(t) E_y(t) \rangle \\
2 \langle E_x(t) E_y(t + t_0) \rangle
\end{pmatrix}
$$
\[
\frac{d}{ds} \begin{pmatrix} \xi \\ \psi \end{pmatrix} = \begin{pmatrix} \text{matrix of intrinsic coefficients} \\ \text{vector of collisions} \end{pmatrix} \begin{pmatrix} \xi \\ \psi \end{pmatrix} + \begin{pmatrix} \xi \\ \psi \end{pmatrix}
\]
Why is wave interference important?

It is directly related to all phenomena of radiation

...continue reading...

Optically, continuous polarization is produced by the scattering of light from electrically charged particles in the environment of the medium.
Pulsar Sounds
Radio signals demodulated into audio signals

<table>
<thead>
<tr>
<th>Pulsar</th>
<th>P (ms)</th>
<th>f=1/P (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B0329+54</td>
<td>714</td>
<td>1.4</td>
</tr>
<tr>
<td>B0950+08</td>
<td>253</td>
<td>3.9</td>
</tr>
<tr>
<td>B0833-45 (Vela)</td>
<td>89</td>
<td>11.2</td>
</tr>
<tr>
<td>B0531+21 (Crab)</td>
<td>33</td>
<td>30.2</td>
</tr>
<tr>
<td>J0437-4715</td>
<td>5.7</td>
<td>174</td>
</tr>
<tr>
<td>B1937+21</td>
<td>1.56</td>
<td>642</td>
</tr>
</tbody>
</table>

Dipolar Fields

MAGNETIC POLES AND THE POLARIZATION STRUCTURE OF PULSAR RADIATION
V. RADHAKRISHNAN AND D. J. COOKE

Comparisons of the polarization structures of PSR B1937+21 at different frequencies lead to the conclusion that pulsar radiation must emanate from the neighborhood of magnetic poles. The most prominent periodicity of this pulsar was not accompanied by any gross changes in its magnetic field structure.
\[
\psi = \psi_0 + \tan^{-1}\left\{ \frac{\sin \alpha \sin (\Omega t - \phi_0)}{\sin \epsilon + [1 - \cos (\Omega t - \phi_0)] \cos \beta \sin \alpha} \right\}
\]

\[\sigma = \beta - \alpha\]

**Equation of field line:**
\[
r(\theta) = R_s \sin^2 \theta / \sin^2 \theta_{\text{FC}}
\]

**Light-cylinder radius:**
\[
r_{LC} = \frac{c}{\Omega} = \frac{c P}{2 \pi}
\]

**Last open field line:**
\[
r_{LC} = r(\pi/2) = \frac{R_s}{\sin \theta_{\text{FC}}}
\]

**Size of magnetic polar cap:**
\[
2\theta_{PC} \approx (2R_s/r_{LC})^{1/2} \\
\approx 1.66 \text{deg} \cdot P^{-1/2} \left( \frac{R_s}{10 \text{ km}} \right)^{1/2}
\]

**Expected pulse width:**
\[
W = \frac{W_t}{P} \approx 2\theta_{PC} \left( \frac{r_{\text{rem}}}{R_s} \right)^{1/2}
\]
From a talk by Joeren Stil, University of Calgary 2009

**Origin and Evolution of Cosmic Magnetic Fields:**
One of Five **Key Science Goals** for SKA

- Radio polarization carries information about magnetic fields
  - in the emitting source
  - in the intervening medium (propagation effect Faraday Rotation)

\[
\Delta \theta = \lambda^2 0.81 \int_{\text{source}} n_e \vec{B} \cdot d\vec{l} = \lambda^2 \text{RM}
\]

---

**The Rotation Measure Sky**

Red: Positive RM (average magnetic field pointing towards observer)  
Blue: Negative RM (average magnetic field pointing away from observer)

Structure on angular scales $< 1^\circ$ to $> 90^\circ$
Polarization of Distant Spiral Galaxies

Effelsberg survey of galaxies in the Local Supercluster (in progress) with M. Krause & R. Beck at MPIfR

Image Dynamic Range in Polarization

Pixel histogram of NVSS Stokes Q Images masked Stokes I sources

Small effect on polarization bias

Strong effect on false detection rate in polarization

Number of polarized sources per square degree
CMB Polarization

Glitches
Spin noise

Differential rotation, superfluid vortices

Interstellar dispersion and scattering

GPS time transfer
Additive noise
Instrumental polarization

Uncertainties in planetary ephemerides and propagation in interplanetary medium

Emission region: beaming and motion
Propagation through the interstellar plasma

Refractive indices for cold, magnetized plasma

\[ n_{t,r} \sim 1 - \frac{\nu_p^2}{2\nu^2} \mp \frac{\nu_p^2\nu_B^2}{2\nu^3} \]

\[ \nu \gg \nu_p \sim 2 \text{kHz} \quad \nu \gg \nu_B \sim 3 \text{Hz} \]

Propagation velocities are frequency dependent:

Phase velocity: \( v_p = \frac{c}{n_{t,r}} \)

Group velocity: \( v_g = \frac{\partial \omega}{\partial k} = \frac{\partial}{\partial k} \left( \frac{kc}{n_{t,r}} \right) \)

Group delay = \( \Delta \text{(Time of Arrival)} \)

\[ t = t_{DM} \pm t_{RM} \]

\[ t_{DM} = 4.15 \text{ ms DM} \nu^{-2} \]

\[ t_{RM} = 0.18 \text{ ns RM} \nu^{-3} \]

Dispersion Measure DM = \( \int ds \, n_e \) units: pc cm\(^{-3}\)

Rotation Measure RM = \( 0.81 \int ds \, n_e B_{||} \) units: rad m\(^{-2}\)

---

A Single Dispersed Pulse from the Crab Pulsar

\( S \sim 160 \times \text{Crab Nebula} \)

\( \sim 200 \text{ kJy} \)

Detectable to \( \sim 1.5 \text{ Mpc} \)

with Arecibo

\[ t_{DM} = 4.15 \text{ ms DM} \nu^{-2} \]

\( \nu \text{ in GHz} \)

\( n_e \text{ in \text{cm}^{-3}} \)

Dispersion measure:

\( DM = \int ds \, n_e(s) \) pc cm\(^{-3}\)
Interstellar scattering from electron density variations

Pulsar velocities >> ISM, observer velocities
Scattering is strong for frequencies < 5 GHz
Electron density irregularities exist on scales from ~100’s km to Galactic scales
Pulse broadening from interstellar scattering:
\[ \tau_d \sim \frac{\text{rms excess path length}}{c} \sim \frac{D\theta_d^2}{2c} \propto \nu^{-4} \]

Low-DM pulsar: \( \tau_d \ll \) pulse width

High-DM pulsar: \( \tau_d \sim \) pulse width

Arecibo WAPP data, Bhat et al 2004
Extra

Parts

- LNA
- Square-Law Detector
- ADC
- Lowpass Filter
- Bandpass Filter
- \( \pi / 2 \)