E & M Questions

1. Consider a conducting disk with radius \( a \) situated in a uniform magnetic field \( B \hat{z} \). The disk surface is perpendicular to the field. A stationary voltmeter is attached via stationary wires with brush contacts, one touching the disk at its center and the other at its outer edge. The disk is spun up to angular frequency \( \Omega \hat{z} \). Calculate the voltage measured if \( B = 10^2 \) G, \( \Omega = 62.8 \) s\(^{-1} \), and the disk radius is \( a = 2 \) cm.

2. Consider the propagation of radio pulses from a radio pulsar at distance \( D = 3 \times 10^{20} \) cm from the earth. The interstellar space contains a low-density of electrons (and ions), \( n_e = 0.1 \) cm\(^{-3} \), and negligible magnetic field. The dielectric constant is \( \epsilon = 1 - \omega_p^2/\omega^2 \), where \( \omega_p \) is the plasma frequency. For radio observations at \( f = 10^9 \) Hz, \( \omega >> \omega_p \). Let \( \tau(\omega) \) be the extra, frequency dependent time-delay for a pulse arriving at the earth, i.e. \( t_{\text{Earth}} = t_{\text{Pulsar}} + D/c + \tau \). Calculate \( \tau \) is s.

3. Calculate the equations of motion for a relativistic particle with charge \( q \) and rest mass \( m \) from the relativistic Lagrangian

\[
\mathcal{L} = -\frac{mc^2}{\gamma} + q\vec{v} \cdot \vec{A} - q\phi \tag{1}
\]

\[
\gamma = \frac{1}{\sqrt{1 - (v/c)^2}} \tag{2}
\]

where \( (\vec{A}, \phi) \) are the vector and scalar potentials and \( \gamma \) is the Lorentz factor. Your answer should be in the form \( d\vec{p}/dt = \ldots \) where \( \vec{p} = m\gamma \vec{v} \).

4. Calculate the motion of a particle with charge \( q \) and rest mass \( m \) starting from rest at \( \vec{r} = 0 \) in constant crossed E and B fields. That is, \( \vec{E} = (E_x, 0, 0) \) and \( \vec{B} = (0, B_y, 0) \) with \( E_x \) and \( B_y \) constant in time and space.