Spectrometer / Spectrograph: Device that calculates a Fourier Transform
To properly represent a sinusoidal component of a signal, you must **sample twice per period**

- Imposes restrictions on the **bandwidth** of the signal you want to sample
  - Nyquist theorem: If your signal has a bandwidth of $\Delta v$, you must sample at $\nu_s = 2\Delta v$
  - (Generally your signal will be “baseband”, so the Nyquist theorem imposes a restriction on the highest frequency component of the signal.)
- Failure to properly sample a signal results in **aliasing**
SCHEMATIC DIAGRAM OF A BASIC DIGITAL SPECTROMETER
MIXING

• The shift theorem for Fourier Transforms allows us to shift our signal in frequency without loss of information.
DIGITAL FILTERING

- Exploits the convolution theorem
- Convolve your time samples with a \( \sim \text{sinc}(x) \) function
- Fourier transform is product of signal and rectangle function
SCHEMATIC DIAGRAM OF A BASIC DIGITAL SPECTROMETER

Continuous (in time and freq):

Periodic <=> Discrete:

Periodic & Discrete <=>
Discrete & Periodic:

\[ F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} \, dt \]

\[ f(t) = \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} \, d\omega \]

\[ f[n] = \sum_{n=0}^{N-1} x[n] e^{i\omega_0 n} \quad \text{where } k = 0, \ldots, N-1 \]
\[ X_k = \sum_{n=0}^{N-1} x_n e^{2\pi i k n / N} \text{ where } k = 0, ..., N - 1 \]
TWO TIME-FREQ UNCERTAINTY RELATIONS

\[ \Delta t_{\text{window}} \Delta f_{\text{channel}} = 1 \]

\[ \Delta t_{\text{sampling}} \Delta f_{\text{bandwidth}} = 1 \]
FFT

- Fast Fourier Transform
- Efficient implementation of the DFT
- Computations $\sim N \log(N)$
- Lengths need to be powers of two

Real data $\rightarrow$ two spikes
Ordering of freq channels
Bias $\rightarrow$ DC bin
$N=256 \quad \Delta \nu \approx 400 \text{ kHz}$

$N=4096 \quad \Delta \nu \approx 24 \text{ kHz}$