Simple Estimates of Pulsar Yields in Galactic Plane Surveys

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Basic Ideas

Given rough estimates of the number of pulsars in the Galaxy, it is useful to know what the expected yield might be for a survey. This document presents a coarse, back-of-envelope type estimate.

How far can we look? Suppose we detect a pulsar with signal-to-noise ratio $S/N$ and we estimate its distance (post facto) from DM as $\hat{D}$. The survey has a threshold $(S/N)_{\text{thresh}}$. Assuming that only inverse-square law influences detection (i.e. not worrying about pulse broadening from instrumentation, dispersion, or scattering), the maximum distance that the source can be detected is

$$D_{\text{max}} = \left[ \frac{S/N}{(S/N)_{\text{thresh}}} \right]^{1/2}. \quad (1)$$

Some known pulsars detected in the PALFA survey have $\hat{D} = \text{few kpc}$ and $S/N > 100$. These are detectable to distances $> 30 \text{ kpc}$, well beyond the Galactic disk.

How many more pulsars of a given type can we expect? Suppose we find one pulsar of interest after analyzing $N_p$ pointings, corresponding to $\Omega_p$ solid angle. How many more such pulsars can we expect if we survey a total solid angle $\Omega_{p,\text{total}}$? In the mean,

$$N_{\text{psr}} = \frac{\Omega_{p,\text{total}}}{\Omega_p} \left( \frac{D_{\text{max}}}{\hat{D}} \right)^3. \quad (2)$$

Number of pointings per pulsar: Suppose there are $N_{\text{psr},g}$ pulsars in a Galactic volume $V_g$. Assume that observations are made in the Galactic plane, where the pulsar density is highest and to distances and in directions that don’t reach the edge of the population. The mean number of pulsars expected in a subvolume that extends to distance $D_{\text{max}}$ in a sampled solid angle $\Omega_s$ is

$$\Delta N_{\text{psr}} = n_{\text{psr}} V_{\text{pointing}} = \frac{f_b N_{\text{psr},g} D_{\text{max}}^3}{3 V_g} \Omega_s, \quad (3)$$

where $f_b$ is the beaming fraction. For ALFA, $\Omega_s = 7\Omega_b$, with $\Omega_b = (\pi/4)\theta_b^2$ where $\theta_b$ is the FWHM of an individual beam ($\sim 3.5 \text{ arcmin}$).

For a uniform disk model for the population, we have $V_g = 2\pi R_g^2 H$, with $R_g =$ galactocentric radius and $H =$ scale height.

The expected number of pulsars per pointing is then

$$N_{\text{psr/pointing}} = \frac{7 f_b N_{\text{psr},g} D_{\text{max}}^3 \Omega_b}{6\pi R_g^2 H} \quad (4)$$

The reciprocal of $N_{\text{psr/pointing}}$ is the number of pointings needed to find one pulsar,

$$N_{\text{pointing/psr}} = \frac{6\pi R_g^2 H}{7 f_b N_{\text{psr},g} D_{\text{max}} \Omega_b} \approx 42 \left( \frac{0.2}{f_b} \right) \left( \frac{10^5}{N_{\text{psr},g}} \right) \left( \frac{5 \text{kpc}}{D_{\text{max}}} \right)^3 \left( \frac{R_g}{8 \text{kpc}} \right)^2 \left( \frac{H}{0.5 \text{kpc}} \right). \quad (5)$$

Canonical pulsars: A beaming fraction of 0.2 is ok, on average. A birth rate of 1/century and a radio-emitting lifetime of 10 Myr yields the nominal $N_{\text{psr},g} = 10^5$. RRATs may imply a larger number but they will be missed in periodicity searches (which are assumed here). The galactocentric radial scale of 8 kpc may be too large but probably not by a factor of two given that pulsars move away from their birth places. A scale height of 0.5 kpc is reasonable, though about 50% of canonical pulsars will escape the Galaxy. Finally $D_{\text{max}} = 5 \text{ kpc}$ is only a crude average. A proper analysis would consider the pseudo-luminosity function as well as a period-dependent beaming fraction. Or, better yet, a true beaming model should be used. Given the uncertainties, the number of ALFA pointings (with 7 beams/pointing) is expected, conservatively, to be a few tens to 100 per pulsar.

Millisecond pulsars: The birth rate is lower by a factor $\sim 100$ while the radio-emitting lifetime is at least 100 times larger, thus yielding approximately the same number of MSPs, $N_{\text{psr}} \approx 10^5$. Other factors are largely the same, though the population, being much older than canonical pulsars, extends to larger $R_g$ with approximately a 0.5 kpc
scale height. The beaming fraction may be somewhat larger than 0.2. The primary difference is that MSPs will have a smaller average detection distance, \( D_{\text{max}} \), perhaps by a factor of two. This will increase the number of pointings to 340 per pulsar, though with factor of two uncertainty at least.

**NS-NS binary pulsars:** These merge at a rate \( \sim 10^{-5.5} \text{ to } 10^{-3.7} \text{ yr}^{-1} \) (3 to 190 per Myr, Kim et al. 2006, astro-ph/0608280). Taking this as the birth rate (which underestimates the BR of all NS-NS binaries, as some will not merge on a Hubble time), and that the radio emitting lifetime of the recycled pulsar in the binary is determined by the merger time scale \( \sim 50 - 200 \text{ Myr} \), typically, the number of NS-NS binaries in the Galaxy \( \sim 50 - 10^{14.6} \) and the number beamed toward us \( \sim 10 - 10^{3.9} \). Using the same values for other parameters in Eq. 5, the number of pointings per recycled NS-NS pulsar becomes \( 10^2 - 10^{1.9} \). Detectability of the un-recycled pulsar in a system has lower probability owing to the radio-emitting lifetime being about \( \times 10 \) shorter. Discovery of the non-recycled pulsar in J1906+0746 in the early stages of the PALFA survey suggests a pointings-per-pulsar value near the lower end of the range.

**How many more pulsars of a given type can we expect (Part II)?** Upon discovery of a single pulsar of interest (e.g. J1903+03) it is tempting to scale up to some total number of similar discoveries, as in Eq. 2. Such estimates are subject to large uncertainties from the small-number statistics. A Bayesian approach brings out these uncertainties.

Let \( x = \) the mean number of pulsars expected per pointing. Then in \( N_p \) pointings the number of pulsars expected is \( xN_p \). Suppose that \( k \) pulsars are in fact discovered in \( N_p \) pointings. Using Poisson statistics and assuming that all pointings are equal in quality, the posterior pdf (probability density function) for \( x \) is

\[
f(x) = f(x|k \text{ pulsars in } N_p \text{ pointings}) = \frac{(xN_p)^k}{k!} e^{-xN_p},
\]

where the left-hand side is the posterior pdf assuming a flat prior on \( x \). Now suppose we make \( M_p \) additional pointings of the same type. How many more pulsars \( M_{\text{psr}} \) can we expect? The conditional pdf of \( M_{\text{psr}} \) is

\[
f(M_{\text{psr}} \text{ in } M_p \text{ pointings}|x) = \frac{(xM_p)_{\text{psr}}^{M_p}}{M_{\text{psr}}!} e^{-xM_p}
\]

and the total pdf, using Eq. 6 as the prior, is

\[
f(M_{\text{psr}} \text{ in } M_p \text{ pointings}) = \int dx f(x)f(M_{\text{psr}} \text{ in } M_p \text{ pointings}|x)
\]

\[
= \frac{M_p^{M_{\text{psr}}N_p^k}}{M_{\text{psr}}!k!} \int_0^{\infty} dx x^{(M_{\text{psr}}+k)} e^{-x(N_p+M_p)}.
\]