What are gravitational waves?

• Oscillations in the gravitational field

• “ripples” in the curvature of spacetime

+ polarization

× polarization

(waves propagating into the page)
What are gravitational waves?

A prediction of general relativity

• Einstein realized that Newtonian gravity had some flaws:

  • instantaneous propagation

  \[ \nabla^2 \Phi(x, t) = 4\pi \rho(x, t) \]

  • dependence on absolute notions of “distance” and “time”

\[ \ddot{r} = -\nabla \Phi(x, t) = -\frac{GM}{r^2} \hat{r} \]
What is general relativity?

• Einstein constructed a new theory of gravity that satisfies the principles of relativity. In this theory:
  
  • Gravity is not a force, but is a manifestation of life in a curved geometry

  • Matter and energy “deform” spacetime; objects then move on this deformed surface
Some manifestations of relativistic gravity:

1. Precession of orbits:

   total precession: 575 arcsec/century
   due to relativity: 43 arcsec/century

   (1 arcsec = 0.000278 deg)
Some manifestations of relativistic gravity:

2. Deflection of light:
Some manifestations of relativistic gravity:

3. The existence of black holes:
Some manifestations of relativistic gravity:

3. The existence of black holes:
Some manifestations of relativistic gravity:

4. The existence of gravitational waves:

Indirect evidence for gravitational waves already exists!

- Timing of the pulses from the Hulse-Taylor binary pulsar (1913+16) showed that the change in the orbital period agrees with the GR prediction.

- Similar measurements from several other pulsar binary systems confirm this.
<table>
<thead>
<tr>
<th>Property</th>
<th>EM</th>
<th>GW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substance</td>
<td>Oscillations of EM fields that propagate through spacetime</td>
<td>Oscillations of spacetime itself</td>
</tr>
<tr>
<td>Sources</td>
<td>Oscillations of microscopic charges</td>
<td>bulk motion of macroscopic masses</td>
</tr>
<tr>
<td>Speed</td>
<td>300,000 km/s</td>
<td>300,000 km/s</td>
</tr>
<tr>
<td>Wavelength $\lambda$</td>
<td>$\lambda \ll L$ (allows imaging of source)</td>
<td>$\lambda \geq L$ (no imaging of source components)</td>
</tr>
<tr>
<td>compared w/ source size L</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Propagation through matter</td>
<td>Significant absorption, scattering, dispersion</td>
<td>Negligible absorption, scattering, dispersion</td>
</tr>
<tr>
<td>What they tell us</td>
<td>Thermo-dynamic state of diffuse matter</td>
<td>Bulk motion of dense concentrations of matter &amp; energy</td>
</tr>
</tbody>
</table>
How will we detect gravitational waves:

Laser interferometers measure displacement of “test-mass” mirrors
How will we detect gravitational waves:

LIGO: Laser Interferometer Gravitational-wave Observatory

Livingston, Louisiana

Hanford, Washington

VIRGO (French-Italian collaboration)

Cascina, Italy

GEO600 (British-German)

Hannover, Germany

[also TAMA (300 m) in Japan]
How will we detect gravitational waves:

**Resonant mass detectors**

*Bar detectors* (Nautilus, Explorer, Auriga, Niobe, Allegro)

*Mini-GRAIL, Leiden University*
How will we detect gravitational waves:

LISA: Laser Interferometer Space Antenna
Sources of gravitational waves: What will we learn?

**Compact binaries**

Inspiralling stellar-mass compact objects (white dwarfs, neutron stars, black holes)
- Test models of binary stellar evolution
- Measure ranges of masses and spins of compact objects
- Probe central engine of gamma-ray bursts (NS/NS, NS/BH)
- Study the equation of state of nuclear matter at ultra-high densities (NS/NS, NS/BH)
- Test the validity of GR in the strong-field, highly non-linear regime (BH/BH)

**Supermassive black hole (SMBH) binaries**
- Learn how SMBH’s grow
- Probe GR with high precision
- Study ejection of BHs from their galaxies

**Extreme-mass-ratio inspirals**
- Census of the compact objects in galactic centers
- Precision map of the spacetime around a BH---test the validity of the mathematical description of BHs
Sources of gravitational waves: What will we learn?

**Individual compact objects**

Core-collapse supernova
- Probe the inner-workings of the explosion mechanism
- Study the nuclear physics at high densities

Continuous sources
- Pulsars w/ small mountains
- Accreting neutron stars
- Fluid instabilities in rotating neutron stars

**Exotic sources?**
- Gravitational wave remnants from the big bang
- Phase transitions in the early universe
- Cosmic strings
- Signatures of extra dimensions

THE UNEXPECTED!
The challenge (experimental):

Gravitational waves are very weak

- a typical source changes the LIGO arms by $\sim 10^{-21}\text{km} \sim 10^{-18}\text{m} \sim 10^{-9}\text{nm}$
- Need very high precision measurements

Noisy environment---background noise obscures the signals

- Seismic noise: including...ocean waves, logging, traffic...
- Gravity gradient noise: including people, cars, wind, tumbleweed...
- Suspension noise: modes of test-mass suspension
- Shot noise, laser noise
- Radiation pressure from laser on the test masses
- Light scattering
- Residual gas in vacuum tube
- Cosmic rays
The challenge (theoretical):

Need a template gravitational wave to extract the physical parameters (masses, spins) from the detected signals.

Compute gravitational waves from a given type of source (eg., compact binary).

Compute the motion of the source.

Solve the Einstein field equations (hard!).
The challenge (theoretical):

**Newton** vs. **Einstein**

**Equations of gravitational field:**

Gravity is a source for gravity (non-linearity)

\[ G_{\mu\nu} [g_{\alpha\beta}(x^\gamma)] = 8\pi T_{\mu\nu} [\rho(x^\gamma), \cdots] \]

1 eq., 1 variable (\( \Phi \)), simple differential operator

\[ \nabla^2 \Phi(x, t) = 4\pi \rho(x, t) \]

Only mass density

Linear differential operator

6 indep. eqs., 6 indep. variables (\( g_{\mu\nu} \)), complicated differential operator; many, many terms...

\[ G_{\mu\nu} [g_{\alpha\beta}] \sim O(g) + O(g^2) + O(g^3) + \cdots \]

Highly non-linear differential operator
The challenge (theoretical):

The 2-body problem

- Simple, exact solution in Newtonian gravity
- No exact solution in general relativity

\[ r = \frac{a(1 - e^2)}{1 + e \cos \varphi} \]

**Exact solutions**
- Only exist for situations with special symmetry
- Eg., single black hole or star

**Perturbation theory**
- Expand about a known, exact solution in the limit that some quantity is small
- Eg., perturb about flat space or a single black hole

**Numerical relativity**
- Solve the equations on a computer
- No symmetries or approximation
- But have to deal with numerical error
Conclusions:

Gravitational waves are oscillations of spacetime curvature generated by rapidly-moving, dense concentrations of mass.

A worldwide network of detectors is trying to detect these waves.

Once detected, we’ll have a new window with which to view (or listen to!) our universe. Gravitational waves will tell us about:

• the distribution, formation, and properties of black holes, neutron stars, and white dwarfs
• the internal structure of neutron stars and supernova explosions
• testing the nonlinear structure of general relativity and the mathematics of black holes
• We’ll learn something unexpected...