# Gravitational waves from neutron stars: recent developments

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The gravitational waves that bathe the Earth presumably do not vary wildly in strength from year to year. However, our very imperfect understanding of their strengths occasionally does change abruptly, prompted either by new astrophysical observations or by new theoretical predictions and discoveries. Such has been the case in the last year for our expectations for periodic gravitational waves from neutron stars, in two different scenarios: (i) accreting neutron stars in low mass X ray binaries (LMXBs), and (ii) hot, young neutron stars in the first year or so after their formation.

*Accreting neutron stars in LMXBs:*

As is well known, rotating neutron stars can radiate via three mechanisms: (i) non-axisymmetry of the star, (ii) non-alignment between the axis of rotation and a principle axis of the moment of inertia tensor, and (iii) excitation of the stars' normal modes. For mechanism (i), the amount of non-axisymmetry can be parameterized by the equatorial eccentricity $\varepsilon_e = (I_{xx} - I_{yy})/I_{zz}$, where $I_{ij}$ is the moment of inertia tensor and the rotation axis is the $z$ axis. Advanced LIGO interferometers can see non-axisymmetric neutron stars out to $\sim 1$ kpc for $\varepsilon_e \gtrsim 10^{-7}/f_{500}$ with 1/3 year integration time, where the rotation frequency is $500 f_{500}$ Hz (Thorne 1998). The likely values of $\varepsilon_e$ for various neutron star populations are highly uncertain. Aside from the millisecond pulsar population which is highly constrained, we know only that $\varepsilon_e \lesssim 10^{-5}$ (Thorne 1998).

Lars Bildsten has recently given fairly convincing theoretical and observational arguments that many LMXBs like Scorpius X-1 should have values of $\varepsilon_e$ of order $10^{-7}$ or larger and should thus be fairly strong sources (Bildsten 1998). First, recent observations by the Rossi X-Ray Timing Explorer satellite indicate that many of the rapidly accreting stars have spin frequencies clustered near 300 Hz. This is somewhat of a puzzle since the accretion would be expected to spin up the stars to much higher frequencies. Bildsten suggests an explanation for this puzzle: that gravitational wave emission is preventing these sources from being spun up any further, i.e., that all the angular momentum being accreted is being radiated into gravitational waves. The limiting angular velocity then scales as the 1/5th power of $\varepsilon_e$ and is thus fairly insensitive to the amount of non-axisymmetry. Second, Bildsten suggests a specific mechanism for generating the required non-axisymmetry: that lateral temperature gradients due to non-uniform accretion over the surface of the star lead (via temperature-dependent electron capture reactions) to lateral density variations in the crust. The resulting estimated values of $\varepsilon_e$ are of the order $10^{-7}$, consistent with what is required.

The wave strengths for these sources can be predicted directly from the the observed X-ray flux and the inferred accretion rate; the amount of non-axisymmetry (quadrupole) is determined by demanding equality of the spin-up and spin-down torques. The strongest source, Sco X-1, is predicted to be detectable with $\sim 3$ years integration with initial LIGO interferometers (Bildsten 1998). Thus, a priority for the early data runs for LIGO and also VIRGO and GEO will be directed searches for periodic signals from known accreting neutron stars.

*Hot young neutron stars – the* r*-mode instability:*

Just over a year ago, Andersson discovered that the $r$-modes of rotating neutron stars are unstable in the absence of viscosity, for all values of the star's angular velocity (Andersson 1998, see also Friedman and Morsink 1998). The instability is driven by gravitational radiation via the

Chandrasekhar-Friedman-Schutz (CFS) mechanism (Chandrasekhar 1970, Friedman and Schutz 1978), and was reviewed by Sharon Morsink in Issue 10 of Matters of Gravity (Morsink 1997). Over the past year a flurry of papers have explored the dramatic astrophysical consequences of the $r$-mode instability. In this review I'll describe these predicted consequences, and summarize the uncertainties and implications. [For a detailed review of instabilities in rotating stars see Stergioulas 1998].

The picture that is emerging is the following. When a neutron star is first formed it is likely spinning at a substantial fraction of its maximum angular velocity. While the star cools from $\sim 10^{11}K$ to $\sim 10^9K$ via neutrino emission over the first few years of its life, the stars' $r$-modes are excited and radiate copious amounts of gravitational radiation, carrying away as much as $0.01M_\odot c^2$ of energy and most of the initial angular momentum of the star (Lindblom et al. 1998, Andersson et al. 1998). When the transition to a superfluid state occurs at $T \sim 10^9K$, the star is left with angular velocity of $\Omega = (0.05 - 0.10)\Omega_{\rm max}$, the exact value being somewhat uncertain. Here $\Omega_{\rm max}$ is the maximum allowed angular velocity. The predicted wave strengths are such that these sources could be seen out to the VIRGO cluster ($r \sim 20$ Mpc) with enhanced LIGO interferometers (Owen et al. 1998). This is quite an exciting prospect since the event rate could be many per year.

This scenario is consistent with the inferred spin after formation of the Crab pulsar of about $0.05\Omega_{\rm max}$, and also (within the uncertainties of the predictions) of the initial spin period of $\sim 7\,{\rm ms}$ of the recently discovered young pulsar PSR J0537-6910 (Owen et al. 1998). It also resolves the observational puzzle that neutron stars seem to be formed with rather small spins despite one's expectation of near maximal initial spins due to conservation of angular momentum during stellar core collapse. It rules out accretion-induced-collapse of white dwarfs as a mechanism for forming millisecond pulsars; millisecond pulsars must form instead via accretion in which the temperature never gets hot enough to trigger the $r$-mode instability. Finally, since it now seems more likely than before that typical stellar core collapses involve rapid rotation rates, it improves the prospects of our detecting supernovae.

Turn now to the assumptions and calculations that underlie these predictions. There are two conditions for a mode in a realistic neutron star to be CFS-unstable: (i) The mode must be retrograde with respect to the star but prograde with respect to distant inertial observers, the classical CFS condition. Not all $r$-modes will satisfy this condition (Lindblom and Ipser 1998), but the dominant $l = m = 2$ $r$-mode will do so. A crucial point is that this condition is satisfied for all values of $\Omega$ for unstable $r$-modes, whereas it is only satisfied at large $\Omega$ for the previously considered $f$-modes. (ii) When one measures the mode's energy in the rotating frame, the amount of energy lost to viscous dissipation per cycle must be less than the amount of energy per cycle that gravitational radiation reaction adds to the mode. In other words, the viscous dissipation timescale must be longer than the instability growth timescale. For the original CFS instability, calculations in Newtonian and post-Newtonian gravity (Lindblom 1995) had shown that these conditions are satisfied for the $l = m = 2$ $f$-mode only in a certain region in the $T - \Omega$ plane (where $T$ is the stellar temperature) with $\Omega \gtrsim 0.9\Omega_{\rm max}$ and $10^9K \lesssim T \lesssim 10^{10}K$, where $\Omega_{\rm max}$ is the maximum angular velocity. The dependence on temperature arises due to the strong dependence of the coefficients of bulk and shear viscosity on temperature. Since neutron stars are at temperatures $\gtrsim 10^9K$ only for the first few years after their formation, and since it was not clear that the initial value of $\Omega$ would be $\gtrsim 0.9\Omega_{\rm max}$, the conventional view was that the CFS instability would probably not be important in practice (see, eg, Thorne 1998). This picture changed when the $r$-mode instability was discovered. Two independent calculations in Newtonian gravity using a slow rotation approximation have indicated that the instability region in the $T - \Omega$ plane is much larger for $r$-modes, extending down to $\Omega \gtrsim 0.05\Omega_{\rm max}$ (Lindblom et al. 1998, Andersson et al. 1998).

For a newly born neutron star, the evolution of the stars angular velocity and of the $r$-mode amplitude was solved for by making the following assumptions (Owen et al. 1998): Assume that only the dominant, $l = m = 2$, $r$-mode is relevant. Assume that the mode amplitude grows due to the instability until it saturates at a value of order unity due to nonlinear effects. [The predictions are not very sensitive to the assumed saturated value of the mode amplitude]. Then, use conservation of angular momentum to solve for the spin down of the star. While these assumptions seem reasonable, it will be important to verify the qualitative predictions by numerical calculations that allow for nonlinear mode-mode couplings, perhaps using post-Newtonian hydrodynamic codes. There are also uncertainties related to the values of the viscosity coefficients and the temperature of the transition to superfluidity; investigation into these issues is continuing. However, the overall picture of rapid spindown seems very robust.

To conclude, it is not often that elegant but somewhat arcane aspects of general relativity (like radiation reaction due to current multipoles) have such dramatic astrophysical and observational consequences. Let us hope for many more such discoveries.

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