Radiation reaction focus session and the 5th Capra meeting

Éanna Flanagan, Cornell
eef3@cornell.edu

One of the activities supported by the newly created Center for Gravitational Wave Physics at Penn State are *focus sessions*. These are intended to bring together a small group of experts in a single, narrowly defined technical topic in order to make progress (see the article by Sam Finn in Issue 19 of Matters of Gravity). One of the first such focus sessions was held at Penn State from May 24 to 30, dedicated to radiation reaction in general relativity, organized by Warren Anderson, Patrick Brady, Sam Finn and myself. It was followed by the fifth Capra Ranch meeting on radiation reaction, from May 31 to June 2.

Focus session on radiation reaction

One of the primary sources for the space-based gravitational wave detector LISA will be the inspiral of compact objects (neutron stars and solar mass black holes) into supermassive black holes. The last year of inspiral of these systems will typically consist of $\sim 10^5$ or so orbits deep in the relativistic regime near the black hole horizon (Finn and Thorne 2000). In order to compute the gravitational wave signal emitted by these systems, it is hoped to compute the deviations from geodesic motion to linear order in the mass ratio, using black hole perturbation theory. While conservation laws can be used to evolve certain classes of orbits (see, eg, Hughes 2000), the treatment of generic orbits will require the computation of the so-called self force that acts on the compact object.

Over the last few years, several research groups have been developing the mathematical and computational tools necessary for computation of the self force; see the accounts of the previous Capra Ranch meetings in Issues 14, 16 and 18 of Matters of Gravity. The foundations for this effort were laid by a pair of papers in 1997 (Mino, Sasaki and Tanaka 1997; Quinn and Wald 1997), which derived a general formal expression for the self force in an arbitrary vacuum spacetime. The challenge has been to translate this expression into a practical computational scheme for orbits in the Kerr spacetime. Such a computational scheme may also be useful for compact objects spiralling into middleweight black holes ($\sim 10^3 M_\odot$) which may be detected by ground based detectors (see the article by Ben Bromley in Issue 14 of Matters of Gravity). While several different computational schemes are being explored, the most developed and most promising contender at the moment seems to be "mode sum regularization", in which one (i) computes using black hole perturbation theory the contribution $f^\alpha_{lm\omega}$ to the self force on a point particle from the mode $lm\omega$; (ii) subtracts from this an analytically computed counterterm involving some regularization parameters; and (iii) sums over all the modes to determine the total self force (see, eg, Barack et. al. 2002 and references therein). This scheme has by now been implemented to compute gravitational self-forces in the Schwarzschild spacetime (Barack and Lousto 2002). The challenge now is to extend the analysis to the Kerr spacetime.

It was to address this challenge that Sam Finn instigated the focus session. We were fortunate to be able to attract most of the key researchers in the field to attend; eighteen people in all were given office space and internet connections for a week. The format and organization of the focus session was significantly different from traditional conferences based around presentations, and was aimed instead at promoting interactions and collaborations among the participants. While one or two black board talks/discussions were scheduled most days, a significant amount of time was left open.

Some of the highlights of the week were as follows. On the first day Yasushi Mino gave a detailed overview of the status of self-force computations, and Scott Hughes reviewed LISA Science objectives. One of the key technical issues that dominated the discussions of the week was the incomplete status of the theory of linear perturbations of the Kerr spacetime. Specifically, one needs to be able to reconstruct the metric perturbation from the Weyl scalars ψ_0 and ψ_4 for general non-vacuum linear perturbations. Bernard Whiting discussed a proposed method of completing the formalism of Chrzanowski (1975) as corrected by Wald (1978), and Amos Ori discussed another possible method. Steve Detweiler talked about how one might deal with the fact that the Teukolsky-Sasaki-Nakamura perturbation formalism does not include the the "$l = 0, 1$" modes which are needed for self-force computations. Another theme was the freedom of choice in the analytically computed counterterms that one uses to renormalize the self force due to a particular mode. Leor Barack discussed the details of one choice of counterterms, and Steve Detweiler discussed another choice that increased the speed of convergence in the sum over modes (Detweiler, Messaritaki and Whiting 2002) based on a definition of a regularized self-field that satisfies the homogeneous wave equation (Detweiler and Whiting 2002).

Eric Poisson discussed the necessity of going beyond the computation of the self-force to calculate the gravitational wave signal, and the problem that linear perturbation theory is strictly speaking insufficient to compute the gravitational wave signal from the inspiralling orbit that includes the backreaction. For the most general contexts, it will be necessary to use second order perturbation theory (Campanelli and Lousto 1999), a daunting prospect! However, in the adiabatic inspiral regime relevant to most of the observations, it seems likely that the linear perturbation formalism will be highly accurate, and there was some discussion of how one might justify this using a two-timescale expansion of the Einstein equations rather than a straightforward perturbation expansion.

One particularly useful talk was Bob Wald's review of the construction of Green's functions via Hadamard expansions, in which he debunked several myths that have appeared in the relativity literature. In particular, the radius of convergence of the Hadamard series can be zero for smooth Lorentzian metrics, and outside of a normal neighborhood one cannot in general compute a Green's function $G(x, x')$ by summing over the geodesics that join x and x'. Eric Poisson discussed how the Mino-Sasaki-Tanaka-Quinn-Wald self-force expression, when specialized to weak fields and slow motions, reproduces the standard post-1-Newtonian results wherein the self force has a conservative part but no dissipative part (Pfenning and Poisson 2002). A separate approach to computing self-forces would be to use a numerical, time-domain Teukolsky code rather than splitting the field into modes. Carlos Lousto reviewed the status of this field of research. Warren Anderson discussed computations with Adrian Ottewill and myself of a local expansion of the tail piece of the metric perturbation, which could be used as a foundation for a different type of regularization method.

There was an enthusiastic consensus by the end of the week that the format had worked very well, and that the intensive discussions that grew out of the blackboard talks were very useful. These discussions spilled over into the coffee breaks, the offices of the participants, and to the picnic hosted by Sam Finn one evening. The meeting was particularly useful because the people involved had already formed a small, closely knit group via the previous Capra Ranch meetings, and most of the group were focusing closely on a specific narrowly defined research area. I expect that many of the discussions at the sixth Capra meeting next summer will involve research projects and collaborations that were germinated at the focus session.

Fifth Capra Ranch Meeting

The primary focus of the meeting was radiation reaction of point particles, as in previous years, but there was also broader, forward-looking focus on radiation reaction effects in general and in particular in numerical relativity and astrophysics. We followed the informal format used in previous meetings where equal time was allotted to talks and discussions.

The meeting opened with a presentation by Amos Ori on a suggested method for computing the metric perturbation in the ingoing radiation gauge from the Weyl scalars ψ_0 and ψ_4 in the Kerr spacetime, based on working in the frequency domain. He described how this method could be used in principle to compute self forces (see Ori 2002 for details). Yasushi Mino described work in progress on a different approach towards computing the self force in Kerr, using the Regge-Wheeler gauge and an expansion in powers of the black hole's spin parameter a.

Leor Barack and Carlos Lousto discussed different aspects of their recent and ongoing work on computing gravitational self forces in the Schwarzschild spacetime (Barack and Lousto 2002), based on the method of Barack *et. al.* (2002). They described the analytic computation of all the regularization parameters needed for a generic orbit, completed computations for radial trajectories that agreed with the ζ-function regularization scheme of Lousto (2000), and ongoing work on circular orbits. Leor also described analytic computations of the large l behavior of the un-renormalized self force; this information was used to improve the speed of convergence of the sum over modes.

Lior Burko described computations of scalar field radiation reaction for particles in circular orbits about Schwarzschild black holes (Burko 2002), which incorporated corrections to the phasing beyond the leading order in an expansion in the mass ratio (corresponding to an accumulated phase correction of order unity during an inspiral). He discussed the fact that such higher order corrections may eventually be necessary for precision astronomy with LISA.

Steve Detweiler described the Green's function decomposition of Detweiler and Whiting (2002). The advantages of this decomposition over earlier decompositions is that the corresponding regularized metric perturbation is a smooth solution of the homogeneous wave equation, and the particle's motion is a geodesic of the regularized perturbed metric. He also described how the use of a particular normal coordinate system defined by Thorne and Hartle (1985) and extended by Zhang (1986) greatly simplifies the local computation of the singular piece of the metric perturbation that is subtracted, and related the construction to the derivation of the self force based on matched asymptotic expansions given in Detweiler (2001). He then described scalar self-force computations for particles in Schwarzschild (Detweiler, Messaritaki and Whiting 2002) and also new gravitational self-force computations, and showed how the use of the new decomposition speeded up the convergence of the sum over modes.

Saturday started with Eric Poisson describing ongoing work with Claude Barrabès in which he defined a coordinate system called retarded normal coordinates, which are closely related to Fermi normal coordinates except that the construction is based null geodesics rather than spacelike geodesics. He showed how the use of these coordinates rather than other types of normal coordinates greatly simplifies the computations in the classic papers of Dirac (1938) and DeWitt and Brehme (1960), and he anticipated that they would also simplify the computations of Mino, Sasaki and Tanaka (1997) and Quinn and Wald (1997).

Next, Manuela Campanelli described a formalism for computing second order perturbations of Kerr black hole in the time domain using the Weyl scalar ψ_4 (Campanelli and Lousto 1999). She described how to obtain second order quantities that are invariant under both coordinate transformations and tetrad rotations, and showed impressive numerical results comparing the second order gravitational waveform from binary black holes in the close limit with the waveform from full numerical relativity and with the first order waveform. She explained that the main roadblock to applying the code to compute self-forces is the necessity to regularize the formally infinite source terms representing the point particle. On a similar note, Karl Martel described numerical work in which he explored replacing a delta function source with a Gaussian profile, in the context of computing the scalar field sourced by a point particle in Schwarzschild. He compared the waveform obtained from the smeared source to that from the exact source for a point particle, for computations in both Schwarzschild coordinates and Painlevé-Gullstrand coordinates. He showed that good agreement between the waveforms was obtained when the width of the Gaussian profile was suitably chosen.

Bernard Whiting then gave a talk in which he discussed a number of issues, including the status of the problem of the $l = 0, 1$ modes in Kerr, the reason why increased smoothness of the regularized metric perturbation gives rise to improved convergence properties of the sum over modes, and analogous phenomena involving convergence and smoothness that arise in LIGO data analysis. He also discussed the prospects for generalizing to the Kerr spacetime the formalism of Lousto and Whiting (2002) for reconstructing metric perturbations from Weyl curvature perturbations in Schwarzschild, in which he emphasized the key role of the algebraically special solutions.

Ian Jones described an ongoing project at Southampton University aimed at including local radiation reaction forces in nonlinear Newtonian and post-Newtonian hydrodynamic codes. He discussed numerical difficulties involved involved in evaluating the local force expressions caused by the large number of time derivatives required. He reviewed a particular formulation of post-1-Newtonian hydrodynamics due to Blanchet, Damour and Schaeffer that eliminates the time derivatives, and which is well adapted to mass quadrupole radiation reaction, and indicated that they planned to use an extension of this formalism being developed by Faye and Schaeffer to incorporate current quadrupole radiation reaction.

Mark Miller described fully general relativistic, 3+1 dimensional simulations of binary neutron star inspirals, using the code described in Font et. al. (2001), and compared the orbital decay rate obtained to the decay rates obtained from post-Newtonian computations. The full GR code evolved the binary for 10 orbital periods. He explained that oscillations observed in the stellar separation suggested that the initial data used actually corresponded to a slightly eccentric binary. He also showed how to use Richardson extrapolation with several runs with different outer boundaries and grid sizes to estimate the computational error. Currently the error in the decay rate is comparable to the decay rate itself, but the errors will improve with time. He also described computations of binding energy curves for binaries obtained using the so-called conformally-flat, quasi-equilibrium approximation, and the orbital decay rates obtained by combining those binding energy curves with the quadrupole energy loss rate. To compute the binding energy curves, he advocated a new prescription in which one subtracts from the total ADM mass of the spacetime the sum of the ADM masses of isolated, *rotating* neutron stars with appropriately chosen angular momenta (rather than isolated non-rotating neutron stars as had been done in the past).

The Capra meeting, like the focus session, was supported by funds from the Center for Gravitational Wave Physics. Thanks are due to Sam Finn for his organizational skills. All of the presentations can be found online at

http://cgwp.gravity.psu.edu/events/Capra5/capra5.html.

References:

Barack, L., and Lousto, C., 2002, gr-qc/0205043.
Barack, L., Mino, Y., Nakano, H., Ori, A., and Sasaki, M., 2002, Phys. Rev. Lett. **88**, 091101 (also gr-qc/0111001).
Burko, L., 2002, gr-qc/0208034.
Campanelli, M., and Lousto, C., 1999, Phys. Rev. D **59**, 124022 (also gr-qc/9811019).
Chrzanowski, P.L., 1975, Phys. Rev. D **11**, 2042.
Detweiler, S., Messaritaki, E., and Whiting, B., 2002, gr-qc/0205079.
Detweiler, S., and Whiting, B., 2002, gr-qc/0202086.
DeWitt, B.S., and Brehme, R.W., 1960, Ann. Phys. **9**, 220.
Dirac, P.A.M., 1938, Proc. R. Soc. (London) **A 167**, 148.
Finn, L.S., and Thorne, K.S., 2000, Phys. Rev. D **62**, 124021 (also gr-qc/0007074).
Font, J.A. *et. al.*, 2002, Phys. Rev. D **65**, 084024 (also gr-qc/0110047).
Hughes, S., 2000, Phys. Rev. D **61**, 084004 (also gr-qc/9910091); Errata Phys Rev D **63**, 049902 (2001) and **65**, 069902 (2002).
Lousto, C.O., 2000, Phys. Rev. Lett. **84**, 5251 (also gr-qc/9912017).
Lousto, C.O., and Whiting, B.F., 2002, Phys. Rev. D **66**, 024026 (also gr-qc/0203061).
Mino, S., Sasaki, M., Tanaka, T., 1997, Phys. Rev. D **55**, 3547 (also gr-qc/9606018).
Ori, A., 2002, gr-qc/0207045.
Pfenning, M.J., and Poisson, E., 2002, Phys. Rev. D **65**, 084001 (also gr-qc/0012057).
Quinn, T.C., and Wald, R., 1997, Phys. Rev. D **56**, 3381 (also gr-qc/9610053).
Thorne, K.S., and Hartle, J., 1985, Phys. Rev. D **31**, 1815.
Wald, R.M., 1978, Phys. Rev. Lett. **41**, 203.
Zhang, X.-H., 1986, Phys. Rev. D **34**, 991.