Physics 216: Problem Set 1

Due Thursday, Feb 5, 2004

1. Reading:

Read chapter 11 of Kleppner and Kolenkow and also sections 12.1 to 12.3 of chapter 12.

2. A 100 m sprinter accelerates from rest at $5\,\mathrm{ms}^{-2}$ until she reaches her maximum velocity of $10\,\mathrm{m\,s}^{-1}$ which she maintains until she crosses the finish line, at which point she decelerates at $2\,\mathrm{m\,s}^{-2}$ to rest.

a. Sketch her worldline on a spacetime diagram.

b. A reporter with a TV camera is running alongside the track at a constant velocity of $5\,\mathrm{m\,s}^{-1}$. Using a Galilean transformation, compute the motion of the sprinter in the inertial frame of the reporter, and sketch the worldline of the sprinter on a spacetime diagram as seen by the reporter. [Assume the reporter crosses the starting line at the same time as the sprinter.]

3. *Synchronization of clocks in an inertial frame:* In lecture we discussed one possible scheme whereby an observer could in principle set the zero-points of a collection of clocks located at the vertices of a rectangular lattice of rigid, identical rods, thereby defining coordinates (t, x, y, z) of an inertial frame. In this problem we'll consider another possible scheme for setting the zero points of the clocks, and show that it produces the same result as the first scheme in Newtonian mechanics, but a different result in special relativity.

The scheme is based on the fact that two clocks can be unambiguously synchronized when they are the same location, but not when they are separated. Suppose that the observer has a large number of identical clocks, all synchronized, at her location. At some instant she shoots these clocks out radially in all directions, each with velocity v relative to her. The clocks are aimed to hit the vertices of the lattice. When one of the moving clocks hits a lattice vertex, the stationary clock at that lattice vertex is synchronized with the moving clock. In this way the observer can define the zero of time for all the clocks in the lattice.

a. Show that in Newtonian mechanics, this method of synchronization produces the same result as the method discussed in lecture.

b. Now consider the situation in special relativity. For simplicity, consider only the motion of one clock, say in the x direction. Let (t, x) be the inertial frame coordinates of the observer, and suppose that the clock is released from $x = 0$ at $t = 0$. Suppose there is a lattice point at a distance $x = x_0$, so that the event $\mathcal{P}$ when the clock hits the lattice point is $x = x_0$, $t = x_0/v$. Now let (t', x') be the inertial frame coordinates of the moving clock. Use the formulae

$$x' = \gamma(x - vt)$$

$$t' = \gamma(t - vx/c^2),$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$ discussed in lecture to show that the time reading on the moving clock at the event $\mathcal{P}$ is $t' = x_0/(\gamma v)$.

c. Deduce that the time $\tilde{t}$ assigned to any event at that lattice point in the new scheme is related to the time t assigned to that event in the old scheme by

$$\tilde{t} = t + \frac{x_0}{v}(\frac{1}{\gamma} - 1).$$

d. Estimate the size of the correction term in part c. for $x_0 = 10$ light minutes (the current distance to Mars) and for $v = 20\,\mathrm{km\,s^{-1}}$ (typical velocity of a spacecraft).

e. Can you invent a scheme for synchronizing the clocks using rays of light? Does your scheme agree with the standard scheme for Newtonian mechanics? For special relativity?

This exercise illustrates that the inertial frame coordinates (t, x, y, z) associated with an observer are defined by a specific operational measurement procedure, and that other, seemingly reasonable procedures whereby an observer could assign times to distant events can produce different answers.