

Physics 216: Problem Set 2

Due Thursday, Feb 12, 2004

Many of the problems on this problem set refer to two inertial frames, a frame S with coordinates (t, x, y, z) , and a frame S' moving with respect to S with velocity v in the x direction, with coordinates (t', x', y', z') . The two sets of coordinates are related by the Lorentz transformation discussed in lecture: $x' = \gamma(x - vt)$, $t' = \gamma(t - vx/c^2)$, $y' = y$, $z' = z$, where $\gamma = 1/\sqrt{1 - v^2/c^2}$. In numerical calculations take $c = 3 \times 10^8 \text{ m s}^{-1}$.

1. Reading:

Read sections 12.4 to 12.6 of chapter 12 of Kleppner and Kolenkow, and also the sections from the book "Spacetime Physics" by Wheeler and Taylor that were handed out in lecture.

2. Properties of Lorentz transformations:

- Show that the limit as $c \rightarrow \infty$ of the Lorentz transformation is the Galilean transformation discussed in lecture.
- Compute the inverse of the Lorentz transformation, i.e., compute t , x , y , and z in terms of the primed coordinates. Show that the inverse transformation has the same form as the original transformation except that v is replaced by $-v$.
- Let \mathcal{P} be the event $t = x = y = z = 0$, and let \mathcal{Q} be the event $t = t_0$, $x = x_0$, $y = 0$, $z = 0$. Compute the spacetime interval $I(\mathcal{P}, \mathcal{Q})$ in the frame S , and also in the frame S' , and show that the same answer is obtained in both cases.

3. The x coordinate of an event \mathcal{P} in the frame S is $x = 5 \times 10^8 \text{ m}$. The coordinates of \mathcal{P} in the frame S' are $x' = 5 \times 10^8 \text{ m}$, $t' = 5 \text{ s}$. Find the velocity v of the frame S' with respect to S .

4. *Time Dilation with π^+ mesons:* A type of particle called a π^+ meson has a half-life of 18 nanoseconds ($19 \times 10^{-9} \text{ s}$). This means that in a given sample of π^+ mesons, half of the particles will decay within 18 nanoseconds as measured in the reference frame in which the particles are at rest. Half of the remainder will decay in the next 18 nanoseconds, and so on.

- In a particle accelerator π^+ mesons are produced when a proton beam strikes an aluminum target inside the accelerator. The mesons leave this target with nearly the speed of light. If there were no time dilation, and if no mesons were removed from the resulting beam by collisions, what would be the greatest distance from the target at which half of the mesons would remain un-decayed?

- b. The π^+ mesons of interest in a particular experiment have a speed that is 0.9978 that of light. By what factor is the answer to part a. increased by the effects of time dilation? That is, by what factor does time dilation allow one to increase the separation between the detecting equipment and the target?

5. Law of composition of velocities:

- a. Suppose that an object is moving with velocity v' in the x direction in the frame S' , so that the equation for its worldline is $x' = v't'$. Using the Lorentz transformation, show that the equation for the worldline of the object in the coordinates of the frame S is $x = \bar{v}t$, where the velocity \bar{v} is

$$\bar{v} = \frac{v + v'}{1 + vv'/c^2}.$$

This is the law of composition of velocities: the object is moving a velocity v' with respect to S' , S' is moving at velocity v with respect to the frame S . The combined effect of these two motions is that the object is moving at velocity \bar{v} with respect to S . Note that this answer differs from the Newtonian answer of $v + v'$.

- b. Show that if $|v| < c$ and $|v'| < c$, then $|\bar{v}| < c$. This explains why it is impossible to achieve speeds faster than the speed of light, no matter how much velocity you attempt to add to a system.