Theoretical Explanations for the accelerating Universe

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The Universe today

- Observations show that the expansion of the Universe is accelerating, which according to general relativity implies the existence of a form of matter with negative pressure (dark energy).

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}[\rho + 3p]
\]

- Acceleration if \( p < -\frac{1}{3}\rho \)

**Dark matter**: well-motivated candidate particles, recent evidence against modified gravity

**Dark energy**: nature, properties unknown
Evidence for acceleration

- Friedmann equation:

\[
\frac{1}{H_0^2} \frac{\dot{a}^2}{a^2} = \frac{\Omega_M}{a^3} + \frac{\Omega_\Lambda}{a^3(1+w)} + \frac{\Omega_R}{a^4} + \frac{\Omega_k}{a^2},
\]

\[1 = \Omega_M + \Omega_\Lambda + \Omega_R + \Omega_k\]

- Cosmic microwave background (standard yardstick)

\[\Omega_R \approx 10^{-4}, \quad |\Omega_k| \leq 0.05\]

- Infer \(a(t)\) from brightness and redshifts of standard candles (Type Ia supernova)
Evidence for acceleration (cont.)

- No Big Bang
- Supernovae
- CMB
- Clusters

$\Omega_\Lambda$, $\Omega_M$, and $w$ parameters for different cosmological models.

Supernova Cosmology Project
Knop et al. (2003)
Spergel et al. (2003)
Allen et al. (2002)

(Spergel et al. 2006)
Explanations for acceleration: categories

- **Cosmological constant**
  - simplest model, agrees with data, theoretical problems

- **Backreaction of perturbations**
  - no new physics required, seems unlikely, not yet ruled out

- **Dynamical dark energy models**

- **Modifications of general relativity**
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Mixed models (generic)
Explanations for acceleration: frameworks

1. Partial classical models
   - for example, \( H^2 = f(\rho) \), or \( c_s = c_s(z), w = w(z) \)

2. Complete classical models, \( S = \int d^4x \ldots \) 

3. Weakly coupled effective field theories

4. “Natural”, weakly coupled field theories

5. UV complete theories, eg string theory models
Simplest model: Cosmological constant

- The case $w = -1$, constant energy density $\rho_\Lambda \sim (10^{-3} \text{ eV})^4$
- **Puzzle:** expect quantum loops to generate a much larger energy density, $\rho_\Lambda \sim E_{\text{cutoff}}^4$
- Unknown higher energy physics can in principle cancel out this contribution, but it requires exquisite fine tuning.
- String theory predicts a multiverse with huge number of different vacua, each with its own $\rho_\Lambda$. Anthropic principle could explain smallness of our $\rho_\Lambda$.
- Problem: life might still evolve if $\rho_\Lambda$ were 1000 times larger.

[Loeb 2006]

- **Puzzle:** The Universe has expanded by 35 orders of magnitude. Why are dark energy and matter comparable right now? Seems to require fine tuning of initial conditions.
Dark energy versus time
One idea for solving the cosmological constant problem is the fat graviton (Zee 2003, Sundrum 2003). The graviton is a composite particle with size $\ell_{\text{grav}}$. This suppresses loop diagrams that contribute to $\Lambda$.

Room for a fat graviton: $20 \, \mu m \lesssim \ell_{\text{grav}} \lesssim 100 \, \mu m$.

Observed value of $\Lambda$: Short distance tests of Newton’s law

Hard to construct explicit model.
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Backreaction of perturbations

\[ g_{ab} = g_{ab}^{(0)} + \varepsilon g_{ab}^{(1)} + \varepsilon^2 g_{ab}^{(2)} \]
\[ \rho = \rho^{(0)} + \varepsilon \rho^{(1)} + \varepsilon^2 \rho^{(2)} \]
\[ u_a = u_a^{(0)} + \varepsilon u_a^{(1)} + \varepsilon^2 u_a^{(2)} \]
Approximate momentum conservation \( k \sim k_1 + k_2 \) forbids contributions to low spatial frequency observables (Hubble expansion rate) from interaction between subhorizon and superhorizon modes.
Can backreaction drive the observed acceleration?

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- Objection: if \( g_{ab}^{(1)} \sim 10^{-5} \), expect \( g_{ab}^{(2)} \sim 10^{-10} \)
  - Other dimensionless numbers present, e.g. \( k_{\text{nonlinear}}/k_{\text{Hubble}} \)
- Objection: data requires \( g_{ab}^{(2)} \sim 1 \), so pert. theory fails
  - Already interesting if pert. theory predicts large effects
Superhorizon perturbations

• Original work used 2nd order pert. theory, complex, subtle errors
• There is a simple counter-argument. Consider a Universe with no subhorizon perturbations: local Taylor series expansions of general solutions of the Einstein-dust equations can be used.

\[
\langle D_L(z, \theta, \varphi) \rangle_{\theta, \varphi} = \frac{z}{H_0} + \frac{1}{2H_0}(1 - q_0)z^2 + O(z^3)
\]

• Observer measures luminosity distance \( D_L \), infers \( q_0 = -\ddot{a}a/\dot{a}^2 \) via

\[
q_0 = -\frac{4\pi}{3H_0^2}\rho + \frac{1}{3H_0^2} \left[ \frac{7}{5} \sigma_{ab}\sigma^{ab} - \omega_{ab}\omega^{ab} \right]
\]

• We obtain

• Negative \( q_0 \) would require large velocity gradients at horizon scale, excluded by observations
Subhorizon perturbations

- Averaged Einstein equations:
  \[ G_{ab}[\bar{g}_{cd}] = 8\pi \bar{T}_{ab} - \langle G^{(2)}[\delta g_{cd}; \bar{g}_{cd}] \rangle \]
  \( \bar{g}_{ab} \equiv \langle g_{ab} \rangle, \ \delta g_{ab} \equiv g_{ab} - \bar{g}_{ab}, \ \bar{T}_{ab} \equiv \langle T_{ab} \rangle \)

- Can effective stress tensor of fluctuations drive acceleration?
- Order of magnitude estimate for perturbation of size \( R \) and mass \( M \) in Newtonian regime

\[ \delta g \sim \frac{M}{R}, \ T_{ab} \sim (\nabla \delta g)^2 \sim \frac{M^2}{R^4} \]
\[ \frac{\Delta M}{M} \sim \frac{T_{ab} R^3}{M} \sim \frac{M}{R} \ll 1 \]

- Smallness confirmed by Newtonian pert. theory computations of Siegel and Fry (2005)
- Post-Newtonian corrections? Unresolved
Subhorizon perturbations (cont.)

- Observer measures $D_L(z) \equiv \langle D_L(z, \theta, \varphi) \rangle$ and infers deceleration using flat FRW relation
  
  $$q(z) = \frac{1 - (1 + z)^2 D_L''(z)}{(1 + z)D_L'(z) - D_L(z)}$$

- Is it possible to measure $q(z) < 0$ in any inhomogeneous dust Universe?
- Is it possible to reproduce observed $q(z)$?
- Do the spectrum of perturbations in our Universe give rise to $q(z) < 0$?
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- Analyzed using Bondi-Tolman models, spherically symmetric dust Universes

(Nambu & Tanimoto 2005; Alnes et. al. 2005; Chuang et. al. 2005; Vanderveld et. al. 2006; Apostolopoulos et. al. 2006; Kai et. al. 2006; Garfinkle 2006)
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- Do the spectrum of perturbations in our Universe give rise to $q(z) < 0$?  Unresolved

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  (Nambu & Tanimoto 2005; Alnes et. al. 2005; Chuang et. al. 2005; Vanderveld et. al. 2006; Apostolopoulos et. al. 2006; Kai et. al. 2006; Garfinkle 2006)

- Requires detailed computations to confirm or refute claim of Kolb et. al. (2005)
Subhorizon perturbations (cont.)

- Bondi-Tolman metric

\[ ds^2 = -dt^2 + \left( \frac{\partial R(r, t)}{\partial r} \right)^2 dr^2 + R(r, t)^2 d\Omega^2 \]

\[ R(r, t) \propto r[t - t_0(r)]^{2/3}, \quad t_0(r) = -\frac{\lambda r^2}{r^2 + D^2} \]

\[ q(z) = \frac{1}{2} + \frac{3}{2} w(z) \Omega_\Lambda(z) \]

(Vanderveld et. al. 2006)
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Dynamical models of dark energy

• Typically do not address cosmological constant problem
• Can address the cosmic coincidence problem
• Typically invoke new fundamental or effective fields

Quintessence:

\[
S = -\int d^4x \sqrt{-g} \left[ \frac{1}{2} (\nabla \Phi)^2 + V(\Phi) \right]
\]

\[
\rho = \frac{1}{2} \dot{\Phi}^2 + V(\Phi), \quad p = \frac{1}{2} \dot{\Phi}^2 - V(\Phi)
\]

\[
\dot{\Phi}^2 \ll V(\Phi) \implies p \approx -\rho
\]

• **Problem**: expect loop corrections to spoil flatness of potential and small mass of scalar field
• **One solution**: scalar field is the size of compact extra dimensions (radion), protected by diffeomorphism invariance

(e.g., Burgess et al. 2006)
Modified Gravity vs. Dark Energy

- Evidence for dark energy *presumes* validity of general relativity. Perhaps, instead, general relativity is *modified* on large scales.
Modified Gravity vs. Dark Energy

• Evidence for dark energy **presumes** validity of general relativity. Perhaps, instead, general relativity is **modified** on large scales.

• How do we decide if a given a modification of the laws of physics involves a modification of gravity? Perhaps ask which side of the Einstein equation is modified, or which term in action is modified:

\[
G_{\mu \nu} = 8 \pi G T_{\mu \nu}
\]

\[
S = \int d^4x \sqrt{-g} \left( \frac{R}{16\pi G} + S_{\text{matter}}[g_{\mu \nu}, \Psi_{\text{matter}}] \right)
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Modified Gravity vs. Dark Energy

• Evidence for dark energy \textit{presumes} validity of general relativity. Perhaps, instead, general relativity is \textit{modified} on large scales.

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• This criterion is in fact ambiguous. Instead, we should ask if there are universal, long-range, 5th forces between macroscopic bodies.
Modified Gravity vs. Dark Energy

Example:

\[ S = \int d^4 x \sqrt{-g} \frac{R}{16\pi G} + S_{\text{matter}}[e^{\alpha(\Phi)}g_{\mu\nu}, \Psi_{\text{matter}}] - \int d^4 x \sqrt{-\bar{g}} \left[ \frac{1}{2} (\bar{\nabla} \Phi)^2 - V(\Phi) \right] \]

\[ \bar{g}_{\mu\nu} \equiv e^{\alpha(\Phi)}g_{\mu\nu}, \quad \bar{\Phi} \equiv f(\Phi), \]

\[ S = \int d^4 x \sqrt{-g} \left[ \frac{A(\bar{\Phi})\bar{R}}{16\pi G} - \frac{1}{2} (\bar{\nabla} \bar{\Phi})^2 - \bar{V}(\bar{\Phi}) \right] + S_{\text{matter}}[\bar{g}_{\mu\nu}, \Psi_{\text{matter}}] \]
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- In this theory, scalar field both acts like quintessence and mediates 5th forces.
- This mixed character is generic, since loop corrections generate matter couplings.
- Solar system tests of gravity (light bending, perihelion precession) require \(|\alpha'(\Phi)| \lesssim 10^{-2}\) if \(V''(\Phi) \ll (\text{A.U.})^{-2}\)
- These theories can arise as effective description of extra dimensions
- Other possible observational signatures: time evolution of effective Newton’s constant (fine structure constant for generalized models)
Modifying gravitational action: a catalog

- Are there successful models that are not “mostly quintessence”?

\[ S[g_{\mu\nu}, \Psi_m] = \int d^4 x \sqrt{-g} \frac{f(R)}{16\pi G} + S_m[g_{\mu\nu}, \Psi_m] \]

- Equivalent to last model with \( \alpha(\Phi) = \Phi / \sqrt{6} \), ruled out by Solar System (Chiba 2003)
- Loopholes: (i) Can make \( V(\Phi_0) \sim m_p^2 H_0^2, \quad V''(\Phi_0) \gg (A.U.)^{-2} \) with fine tuning (Nojiri & Odintsov 2003) (ii) In special cases nonlinearities in potential can be important, acts like chameleon field models

\[ S[g_{\mu\nu}, \Psi_m] = \int d^4 x \sqrt{-g} \frac{f(R, R_{\mu\nu} R^{\mu\nu}, R_{\mu\nu\lambda\sigma} R^{\mu\nu\lambda\sigma})}{16\pi G} + S_m[g_{\mu\nu}, \Psi_m] \]

Modifying gravitational action: a catalog

- Ruled out; predicts modifications of particle physics at energy scale
  \[ \sim \sqrt{H_0 M_p} \sim 10^{-3} \text{ eV} \quad (\text{EF 2005}) \]

- Some successful models. Equivalent to tensor bi-scalar model similar to the mixed models discussed earlier.

- Some interesting models based on extra dimensions do not have a simple effective 4-dimensional description, eg DGP model
Modifying gravitational action: a catalog

- The case $\alpha(\Phi) = 0$, k-essence theories, solve coincidence problem

- When in addition have $F(K, \Phi) = F(K)$ with local minimum, get ghost condensate model (Arkani-Hamed et al. 2003). Cosmological attractor. String theory version found (Mukohyama 2006)

- Well posed initial value formulation and avoiding ghosts require $F, K > 0$ and $F, K + 2KF, KK > 0$ (Armendariz-Picon 2005)

- Superluminal propagation for $F, KK > 0$. Not a problem classically (Bruneton 2005) but may be in QFT (Arkani-Hamed et al. 2006)

- Novel gravitational effects for $\alpha(\Phi) \neq 0$. Corrections to Newton’s law, violations of equivalence principle (EF & Vanderveld 2006)
Observational probes of dark energy

- Probes of the expansion history of the Universe
- Probes of the growth of perturbations
- Precision tests of general relativity
- Measurements of time evolution in fundamental constants of nature
- Specific observational windows (i) Supernovae (ii) Measurements of numbers of clusters using CMBR (iii) Weak gravitational lensing (iv) Baryon acoustic oscillations
Conclusions

- The backreaction explanation for the acceleration of the Universe has not been ruled out, but seems unlikely.
- If backreaction does not work, explaining the acceleration requires new fundamental physics.
- The dark energy might be a cosmological constant. We may never be able to explain its tiny size.
- The dark energy may be dynamical. There are a variety of theoretical scenarios and a variety of observational windows.