Implementing Mino’s prescription for computing waveforms from compact objects inspiralling into black holes

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Motivation

- LISA should observe $\sim 1000$ inspirals of compact objects ($\mu \sim 1 - 10M_\odot$) into massive black holes ($M \sim 10^6M_\odot$) (Gair et al. 2003). Last $\sim 1$ yr of inspiral will contain $\sim M/\mu \sim 10^5$ cycles of waveform (Finn and Thorne, 2000) in the relativistic regime $v/c \sim 1$.

- Many scientific payoffs:
  1. Measure BH masses and spins to accuracy $\sim 10^{-4}$ (Poisson 1994, Barack & Cutler 2004); constrain growth history (mergers versus accretion) of the black holes (Hughes & Blandford 2003).
  2. Learn about central parsec of galactic nuclei from measured event rate and distribution of inspiralling objects masses.
  3. Precise test of general relativity in the strong field regime. Measure multipole moments of central object (Ryan 1995, 1997), unambiguous identification as BH

- All of these require templates with fractional phase accuracy $\sim 10^{-5}$. 
Current methods of computing templates

Use of post-Newtonian methods: Yields crude waveforms that have been already used to roughly scope out LISA’s ability to detect inspiral events (Gair et al. 2004) and to measure the waveform’s parameters (Barack & Cutler 2004).

Use of conservation laws: Use fluxes of energy and $z$-component of angular momentum to infinity and down the black hole to infer adiabatic evolution of orbit; restricted to equatorial and circular orbits (Cutler, Kennefick & Poisson 1994; Shibata 1994; Glampedakis & Kennefick 2002; Hughes 2000, 2001).

Direct computation of the self-force: Formal expression known (Mino, Sasaki & Tanaka 1997, Quinn & Wald 1997). Practical computational scheme in Kerr is difficult; much work over last few years.

Adiabatic waveforms

- Orbital periods $\sim M$; radiation reaction time $\sim M^2/\mu = M/\varepsilon$, where $\varepsilon = \mu/M$. Orbital evolution governed by

$$\frac{d^2 x^\alpha}{d\tau^2} + \Gamma^\alpha_{\beta\gamma} \frac{dx^\beta}{d\tau} \frac{dx^\gamma}{d\tau} = \varepsilon \left[ a^\alpha_{0,\text{diss}} + a^\alpha_{0,\text{cons}} \right] + \varepsilon^2 \left[ a^\alpha_{1,\text{diss}} + a^\alpha_{1,\text{cons}} \right] + O(\varepsilon^3),$$

where $\vec{a}_0 = P \cdot \nabla \cdot (h_{\text{ret}} - h_{\text{sing}})$, and $\vec{a}_1$ is unknown.

- Orbital phase can be expanded using two-time expansion as $\Phi(t) = \frac{1}{\varepsilon} \left[ \Phi_0(t, \varepsilon t) + \varepsilon \Phi_1(t, \varepsilon t) + O(\varepsilon^2) \right]$.

- Adiabatic waveforms are those obtained using leading order orbit $\Phi_0$ and using (modified) linear Einstein equation. Instantaneously accurate to $O(\varepsilon)$, cumulative phase errors $\sim O(1)$.

- Orbits characterized by $E = \xi u^\alpha$, $L_z = \psi u^\alpha$, $Q = K_{\alpha\beta} u^\alpha u^\beta$. Sufficient to know $\langle \dot{E} \rangle$, $\langle \dot{L}_z \rangle$, $\langle \dot{Q} \rangle$. Conservation-law waveforms are adiabatic.

- Mino (2003) has shown how to obtain adiabatic waveforms for generic orbits: $\langle \dot{Q} \rangle = 2\varepsilon \langle K_{\alpha\beta} u^\alpha a^\beta_0 \rangle = 2\varepsilon \langle K_{\alpha\beta} u^\alpha a'_0 \rangle$ where $\vec{a}'_0 = P \cdot \nabla \cdot (h_{\text{ret}} - h_{\text{adv}})/2$. 
Accuracy of adiabatic waveforms

- In slow-motion limit $v/c \ll 1$, post-adiabatic correction terms can be identified in post-Newtonian expressions. Using post-3.5-Newtonian waveforms to compute the phase error, minimized over time of arrival and initial phase, for a $10M_\odot, 10^6 M_\odot$ inspiral gives:

- Adiabatic waveforms are likely good enough for signal detection [phase coherence requirement $\sim 3$ weeks (Gair et al. 2004)], but not for parameter extraction (phase coherence requirement entire signal).
Implementing Mino’s prescription

- Use expansion of Gal’tsov (1992) of radiative Greens function in terms of modes. Use radiation gauge (expression for $<\dot{Q}>$ is gauge invariant). Evaluate time average using integral over torus in phase space (Drasco & Hughes 2004), obtain:

$$
\langle \dot{E} \rangle = \sum_{\Lambda} |Z^{H}_{\Lambda}|^2 + |Z^{\infty}_{\Lambda}|^2
$$

$$
\langle \dot{L}_{z} \rangle = \sum_{\Lambda} \frac{\omega_{mkn}}{m} \left[ |Z^{H}_{\Lambda}|^2 + |Z^{\infty}_{\Lambda}|^2 \right]
$$

$$
\langle \dot{Q} \rangle = \sum_{\Lambda} (\tilde{Z}^{H}_{\Lambda})^*Z^{H}_{\Lambda} + (\tilde{Z}^{\infty}_{\Lambda})^*Z^{\infty}_{\Lambda},
$$

where $\Lambda = (l,m,k,n)$.

- Amplitudes $Z_{\Lambda}^{H}, Z_{\Lambda}^{\infty}$ computed by existing Drasco-Hughes code. New amplitudes $\tilde{Z}_{\Lambda}^{H}, \tilde{Z}_{\Lambda}^{\infty}$ have to be added to code. Both sets of amplitudes are given by integrals over tori in phase space.


Conclusions

• There are no obstacles remaining to prevent computation of adiabatic inspiral waveforms for generic orbits.

• These waveforms will likely be accurate enough for signal detection with LISA.

• Analysis of detected signals and parameter extraction will require going beyond this approximation and using the local self-force. This remains a significant challenge.