Lecture #3: Orbits and Gravity

- Laws of Planetary Motion:
  - Kepler's Laws.
  - Newton's Laws.
- Gravity.
- Planetary Orbits.
- Spacecraft Orbits.

The Main Point

Motions of planets, moons, and asteroids can be very accurately predicted because of the underlying laws of planetary motion and gravity discovered by Kepler and Newton.

Laws of Planetary Motion

- Ptolemaic (geocentric) and Copernican (heliocentric) systems made specific predictions about where the planets would be over time.
- What was needed was high quality data to test these competing models.
- Tycho Brahe (1546-1601):
  - 20+ years of careful measurements.
  - Best data available, and pre-telescopic!
Laws of Planetary Motion

- Tycho provided the data, but his apprentice provided the key new model.
- Johannes Kepler (1571-1630)
  - Mathematician, theorist.
  - Tycho's data weren't fit well by models of Ptolemy or Copernicus.
  - Worst case: Mars. WHY?
- Kepler's 20 years of studies led him to discover three Laws of Planetary Motion.

Kepler's First Law

- Each planet moves about the Sun in an orbit that is an ellipse, with the Sun at one focus of the ellipse.

Kepler's Second Law

- The straight line joining a planet and the Sun sweeps out equal areas in space in equal time.

Kepler's Third Law

- The squares of the planets' periods of revolution are in direct proportion to the cubes of the semimajor axes of their orbits:
  \[ p^2 = a^3 \]
  where:
  - \( p \) = orbit period in years
  - \( a \) = semi-major axis, in AU
    (1 AU = 150 million km) ("Astronomical Unit")
Planetary Motions

• Kepler's discoveries were spectacular!
• BUT... They are empirical laws only.

• WHAT FORCES control this behavior?

• The answer was provided by Isaac Newton.

Newton and Gravity

• Isaac Newton (1643-1727)
  – Mathematician, philosopher.
  – Synthesized observations and results of Tycho, Kepler, and Galileo.
  – Invented calculus!
  – Formulated three Laws of Motion for all objects (not just planets).
  – Discovered the Universal Law of Gravity to explain the motions of planets (and everything else!)
  – Provided the framework for the later telescopic discovery of Neptune, based on gravitational perturbations of Uranus’ orbit.

Newton's Laws of Motion

• First Law:
  – An object at rest remains at rest. An object in motion remains moving uniformly in a straight line unless acted on by an external force.

• Second Law:
  – The change of motion of a body (acceleration) is proportional to the force acting on it (F = ma).

• Third Law:
  – To every action, there is an equal and opposite reaction.

Newton's Laws of Motion

• Newton's 1st law really says momentum is conserved.
  – Momentum = mass times velocity.
  – For angular motion, momentum = mass times velocity times distance from the axis of rotation.
Newton's Laws of Motion

- Conservation of Angular Momentum is a fundamental concept in planetary science, to which we will return many times...

Other Fundamental Properties

- **Mass:**
  - The amount of "stuff" in an object ("stuff" means protons and neutrons and other subatomic particles) [grams].
  - Mass ≠ Weight! Weight describes the force acting on mass.

- **Volume:**
  - The physical space occupied by an object [cm³].

- **Density:**
  - Mass / Volume = Mass per unit Volume [g/cm³]
  - "How tightly packed".

Some typical densities

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (g/cm³)</th>
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<tbody>
<tr>
<td>Neutron Star</td>
<td>~ 10¹⁵</td>
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<tr>
<td>Gold</td>
<td>19.3</td>
</tr>
<tr>
<td>Mercury</td>
<td>13.6</td>
</tr>
<tr>
<td>Lead</td>
<td>11.4</td>
</tr>
<tr>
<td>Iron</td>
<td>7.9</td>
</tr>
<tr>
<td>Earth (bulk)</td>
<td>5.6</td>
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<tr>
<td>Typical Rock</td>
<td>2.5</td>
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<tr>
<td>Water</td>
<td>1.0</td>
</tr>
<tr>
<td>Wood</td>
<td>0.8</td>
</tr>
<tr>
<td>Styrofoam peanut</td>
<td>0.1</td>
</tr>
<tr>
<td>Comet tail</td>
<td>~ 10⁻¹⁵</td>
</tr>
<tr>
<td>Universe</td>
<td>~ 10⁻³⁹</td>
</tr>
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</table>

Gravity: The Tie that Binds

- Newton showed that gravity is the force that bends the planets' straight line paths into ellipses.
- More generally, he showed that gravity is a fundamental attractive force among all bodies everywhere in space.
- Gravity obeys an inverse square law:
  - Gravitational force = GM₁M₂ / R².
  - M₁, M₂ are two masses, R is their separation.
  - G is a universal constant (Appendix A).
Kepler Revisited

- Newton found that Kepler's third law is actually a simplified version of a more exact expression:
  - If $p$ in years, $a$ in AU, and $M$ in solar masses, then $p^2 \cdot (M_1+M_2) = a^3$
  [notice: if $M_1=1$ and $M_2 \ll M_1$ then $p^2 \approx a^3$]
- A Powerful Result!
  - If we can simply observe the period of rotation of one object about another (a planet around the Sun, a satellite around Jupiter, ...) and their separation, we can deduce the mass of the system!

Planetary Orbits

- Generally circular.
- Most in same plane.
- Gap between Mars, Jupiter:
  - Asteroid belt.
  - Why there?
- Many asteroids have high $e$.
- Most comets have high $e$, $i$.
- Is this arrangement typical?

<table>
<thead>
<tr>
<th>Object</th>
<th>$a$ (AU)</th>
<th>$e$</th>
<th>$i$ (°)</th>
<th>$l$ (°)</th>
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<tbody>
<tr>
<td>Mercury</td>
<td>0.39</td>
<td>0.24</td>
<td>0.21</td>
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<td>Venus</td>
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<td>0.62</td>
<td>0.01</td>
<td>3.4</td>
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<td>Earth</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.0</td>
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<tr>
<td>Mars</td>
<td>1.52</td>
<td>0.88</td>
<td>0.00</td>
<td>1.85</td>
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<td>Ceres</td>
<td>2.77</td>
<td>4.00</td>
<td>0.08</td>
<td>10.6</td>
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<tr>
<td>Jupiter</td>
<td>5.20</td>
<td>1.90</td>
<td>0.05</td>
<td>1.31</td>
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<td>Saturn</td>
<td>9.54</td>
<td>2.95</td>
<td>0.06</td>
<td>2.40</td>
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<td>9P/Halley</td>
<td>17.9</td>
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<td>Uranus</td>
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<td>84.1</td>
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<td>Neptune</td>
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<td>346.8</td>
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<td>1.70</td>
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<td>44.2</td>
<td>359.4</td>
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<tr>
<td>C/ Hyakutake</td>
<td>1003</td>
<td>32,000</td>
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<td>125</td>
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(Appendix E)
Many more objects out there!
- 1 million+ asteroids?
- 1 trillion+ comets?
- The solar system does not end at Neptune!
- More details: Lec. 32...

Summary

- **Kepler**
  - Discovered three empirical laws of planetary motion.
- **Newton**
  - Discovered three generalized laws of motion.
  - Derived the basics of gravitational theory.
- Motions of planets, moons, asteroids, and comets can be very accurately predicted!
- These discoveries laid the foundation for the current Golden Age of spacecraft exploration.

Next Lecture...

- Seasons.
- Timekeeping.
- Calendars.
- Readings for next week:
  - Chapters 2.2 and S1.