Impulsive VLBI Jet Formation and High-Energy Gamma-Ray Emission in AGN

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A model is developed for the time dependent electromagnetic – radio to gamma-ray – emissions from AGNs based on the acceleration and creation of leptons at a propagating discontinuity or front of a Poynting flux jet. The front corresponds to a discrete relativistic jet component as observed by VLBI. Equations are derived for the number, momentum, and energy of particles in the front taking into account synchrotron, synchrotron self Compton (SSC), and inverse Compton processes as well as photon-photon pair production. The apparent synchrotron, SSC, and inverse Compton luminosities as functions of time are determined.

1. Introduction

New understanding of the nature of Active Galactic Nuclei (AGNs) has come from the discovery of the high energy gamma-ray radiation in the range 50-2000 MeV by the EGRET instrument on the Compton Gamma Ray Observatory (e.g., [1]). This radiation is observed from a sub-class of AGNs termed blazars, which include Optically Violently Variable (OVV) quasars and BL Lac objects and which show strong variability in all wavebands from radio to gamma. Many of the objects reveal superluminal jets in VLBI maps which indicate that we observe matter of the jet pointed nearly towards us and moving with relativistic speed.

Prior to the Compton Observatory measurements, prediction of strong, collimated gamma ray emission from AGN relativistic jet sources was made by the electromagnetic cascade model of Lovelace and Burns [2,3]. More recently, a number of theoretical models have been developed to explain the observed gamma ray emission of AGNs (see review [4]). In most of the models the gamma ray radiation is ascribed to inverse Compton scattering of relativistic electrons and possibly positrons (Lorentz factors $\gamma \sim 10^2 - 10^3$) of a jet having relativistic bulk motion (Lorentz factor $\Gamma \sim 10$) with soft photons (energies $\sim 1-10^2 eV$). The soft photons can arise from the synchrotron emission of the relativistic electrons in the jet as in the synchrotron self Compton (SSC) models [5,6] or from the direct or scattered thermal radiation from an accretion disk [7-10].

Here, we propose that the main driving force for the observed superluminal jet components is a finite amplitude discontinuity in a Poynting flux jet. A rapid change in the Poynting jet outflow from a disk can result from implosive accretion in a disk with an ordered magnetic field [11]. Propagation of newly expelled EM field and matter from the disk with higher velocity than the old jet leads to the formation of a pair of shock waves as in the case of non-relativistic hydrodynamic flow [12].

2. Theory

We use an inertial cylindrical coordinate system $(r, \phi, z)$ with the origin at the black hole’s center and the $z-$axis normal to the accretion disk as shown in Figure 1. This is referred to as the 'lab frame’. We consider that Poynting flux jets propagate symmetrically away from the disk in the $\pm z$ directions, but focus our attention on the ap-
proaching $+z$ jet. A Poynting flux jet is self-collimated with energy, momentum, and angular momentum transported mainly by the electromagnetic fields [13]. A steady Poynting jet can be characterized in the lab frame by its asymptotic ($z \gg r_o$) magnetic field \( B_\phi = -B[r_o/r_j(z)] \), and electric field \( E_r = -(v/c)B[r_o/r_j(z)] \) at the jet’s edge, \( r = r_j(z) \), where \( r_o \) is the jet’s radius at \( z = 0 \), \( v = \text{const.} \) is the jet’s axial velocity, and \( B \) is the lab frame field strength at \( z = 0 \).

The jet plasma consists of both ions and leptons (electrons and positrons) with the ratio of leptons to ions \( f_{li} \). The initial jet radius is taken to be \( r_o = 6GM/c^2 \), where \( M \) is the black hole mass. The energy flux (luminosity) of the $+z$ jet is the Poynting flux \( L_j = vB^2r_o^2/8 = \text{const.} \).

We propose that the Poynting jet from the disk changes abruptly at time \( t = 0 \). That is, the jet parameters change from values with subscript (1) to subscript (2) at \( t = 0 \), \( (B_1, n_{i1}, v_1, f_{li1}) \rightarrow (B_2, n_{i2}, v_2, f_{li2}) \). In actuality the change will be with a time scale determined by the disk dynamics as in the implosive accretion model of Lovelace et al.[11]. In the present work we consider \( B_1/B_2 < 1 \), \( \nu = n_{i1}/n_{i2} \), \( v_1 < v_2 \), \( f_{li1} = f_{li2} \), where \( n_i \) is the number density of ions. The change in the jet parameters produces a 'front' which propagates outward as indicated in Figure 1. The front may involve a pair of shock waves, one for the incoming old jet matter and the other for the incoming new jet matter. We let \( Z(t) = z(t)/r_o \) denote the dimensionless axial distance of the front. We also use lab time \( T = t/(r_o/c) \), speed of the front \( U(T) = dZ/dT = (dz/dt)/c \), and bulk Lorentz factor \( \Gamma = (1 - U^2)^{-\frac{1}{2}} \). The time measured in the front frame is \( T' = t'/(r_o/c) \) with \( dT' = dT/\Gamma(T) \). The initial values at \( T = T' = 0 \) are \( Z = 0 \) and \( U = V_1 \). We also use the time \( T'' = t''/(r_o/c) \) measured by a distant observer oriented at an angle \( \theta_{obs} < \pi/2 \) to the $z$-axis; \( dT'' = dT[1 - U \cos(\theta_{obs})] \).
**Number of particles:** The continuity equation for ions in the front frame is

\[
\frac{\partial n'_i}{\partial t'} = - \frac{\partial (n'_i v')}{\partial z'},
\]

(2.1)

where \(n'_i, v',\) and \(t'\) are the number density, velocity, and time in the front frame. We integrate this equation over a cylindrical 'pill box' of radius \(r > r_j(z)\) and axial length \(\Delta z'\) and use the Lorentz transformations to obtain an equation for the total number of ions in the front \(N_i(T)\),

\[
\frac{dN_i}{dT} = N_{io} \left[ \Delta V_2 H(\Delta V_2) + \nu \Delta V_1 H(\Delta V_1) \right].
\]

(2.2)

Here, \(N_{io} \equiv \pi r^2 n_{i0}(z) r_0 = \pi r^3 (n_{i0})_{z=0}\) is a constant by our earlier assumptions. Also, \(H(x) = 1 \text{ for } x > 0\) and \(H(x) = 0 \text{ for } x < 0\). If \(\Delta z'\) were a constant, then we would have \(\Delta V_1 = U - V_1\) and \(\Delta V_2 = V_2 - U\), where upper case quantities are dimensionless. However, the plasma in the front expands freely with sound speed \(C'_s\) (normalized by \(c\)) so that \(\Delta Z' = \Delta Z'_0 + 2 \int_0^{T'} dT'' C'_s(T'')\), and this results in modified expressions for \(\Delta V_{1,2}\).

In the radial direction the plasma also expands freely with sound speed \(C''_s\). Magnetic pinching of the front is estimated to be small. Note that the number density of ions in the front frame is simply \(n'_i = N_i / (\pi r^2 R^2 \Delta Z')\). The electric field in the front frame \(|E'|\) is small compared with \(|B'|\).

If \(f_{i1} = f_{i2}\) and there is no \(e\pm\) pair production in the front, then the total number of leptons in the front is simply \(N_l = f_{ii} N_i\). However, in the general case considered here,

\[
\frac{dN_l}{dT} = N_{lo} \left[ f_{i12} \Delta V_2 H_2 + f_{i12} \nu \Delta V_1 H_1 \right] + \frac{1}{\Gamma} \left( \frac{\delta N_l}{\delta T'} \right)_{e\pm},
\]

(2.3)

where the \(H's\) are the same as in equation (2.2). The final term represents the photon-photopon pair production, and is discussed by Romanova and Lovelace [14]. Electron-positron recombination is negligible for the conditions considered.

**Momentum conservation:** In the front frame,

\[
\frac{\partial T'_{0z}}{\partial t'} = - \frac{\partial T'_{zz}}{\partial z'} + grav + rad,
\]

(2.4)

where \(T'_{0z}\) and \(T'_{zz}\) are components of the energy momentum flux density tensor in the front frame, and 'grav' and 'rad' denote gravitational and radiative force contributions not included in \(T'_{ij}\). We integrate (2.4) over the same pill box to obtain

\[
(N_i m_i \gamma_i + N_l m_e \gamma_l) \Gamma^3 \frac{dU}{dT} = - \pi r^2 r_0 \frac{T'_{zz}}{c^2} + \frac{r_o}{c^2} \int (grav + rad) .
\]

(2.5)

\(\gamma_i\) and \(\gamma_l\) denote averages over the ion and lepton distribution functions in the front frame. In this frame the distributions are assumed isotropic and that for electrons is assumed the same as for positrons. The initial values of \(\gamma_i\) and \(\gamma_l\) are considered to be close to unity. A Lorentz transform gives \((T'_{zz})_s = \Gamma^2 (T_{zz} - 2UT_{oz} + \U^2 T_{oo})_s\) where \(s = 1, 2\). The lab frame components of the energy-momentum tensor for a Poynting flux jet are

\[
T'_{oo} = T_{zz} = \frac{E_r^2 + B^2_\phi}{8\pi} = (1 + \V^2)_s \left( \frac{B^2}{8\pi} \right)_s, \quad T_{oz} = \frac{E_r B_\phi}{4\pi} = 2\V_s \left( \frac{B^2}{8\pi} \right)_s.
\]

(2.6)

where \(B = B_\phi\) at the edge of the jet. Thus, in equation (2.5) we have

\[
- \frac{\pi r^2 r_o}{c^2} \left[ (1 + \U^2)(1 + \V^2) - 4\U \V \right] - \frac{r_o^2 B^2_\phi}{8\pi} \left[ (1 + \U^2)(1 + \V^2) - 4\U \V \right].
\]

(2.7)
where $b^2 \equiv (B_1/B_2)^2$. The terms in the square brackets involving $V_2$ represent the push from the new Poynting jet, while those involving $V_1$ are for the push in the opposite direction from the old Poynting jet. Note that $r^2B^2_{z,2} = const = r^2_{o,2}(B_2)^2_{z,2=0}$. A small modification of equation (2.7) similar to that of (2.2) is required to account for the free expansion of the plasma of the front.

Dividing equation (2.5) by $\Delta M_0 = N_i(0)m_i + N_i(0)m_e$ gives

$$\left(\frac{N_i m_i \tilde{r}_i + N_i m_e \tilde{r}_e}{\Delta M_0}\right) \Gamma^3 \frac{dU}{dT} = \mu \Gamma^2 \left\{ \left[ \ldots \right] \right\} + \frac{r_o}{\Delta M_0 c^2} \int (grav + rad), \tag{2.8}$$

where $\mu \equiv r_o^3(B_2)^2_{z=0}/(8\Delta M_0 c^2) \gg 1$ is a dimensionless measure of the strength of the Poynting jet. The brackets $\{\}$ denote the same quantity as in equation (2.7).

The gravitational force in equation (2.8) can be written as $\int grav = -GM(N_i m_i + N_i m_e)/(r_o^2 + z^2)$.

The radiative force depends in general on the geometry and energy distribution of the background radiation field of the central region of the AGN. Dermer et al. [7] consider the case where the radiation comes from the disk, while Sikora et al. [9] argue that the radiation field is from disk radiation scattered by a distribution of clouds orbiting the central object. We adopt a rough version of the radiation field of Sikora et al. The average photon energy is denoted $\tilde{\epsilon}_ph$, the total luminosity $L_{ph}$, and the characteristic radius of the spatial distribution $r_{ph}$. The force due to this radiation field is $\int rad = (N_i \sigma_T) L_{ph} \Delta \phi \mathcal{F}_T/\pi(r_{ph}^2 + z^2)$.

$\mathcal{F}_T = \Gamma^2[\cos(\theta_{ph}) - U]/\cos(\theta_{ph}) - U$ is the Doppler factor which accounts for the change in the flux and the change in the $z$ momentum of the photons between the lab and front frames, $\cos(\theta_{ph}) = z(r_{ph}^2 + z^2)^{-\frac{1}{2}}$.

$\sigma_T$ denotes the Thomson cross section. The contribution of leptons with $\gamma > m_ec^2/\tilde{\epsilon}_{ph}$ to $(N_i \sigma_T)$ in the radiative force is reduced in that the Klein-Nishina cross section applies. $\mathcal{F}_T = (1 - e^{-\tau_T})/\tau_T$ with $\tau_T = n_i' r_o R \sigma_T$ the Thomson optical depth of the front.

Energy conservation: In the front frame,

$$\frac{\partial T_{o,0}'}{\partial t'} = -\frac{\partial T_{o,2}'}{\partial z'} - syn - ssc - com, \tag{2.9}$$

where the last three terms represent the energy loss rates due to synchrotron radiation, inverse Compton scattering off of synchrotron photons, and inverse Compton scattering off of the above mentioned background photons. Following our previous method, we integrate (2.9) over the volume of the front to obtain

$$\Gamma \frac{d}{dT} \left[ N_i (\tilde{\gamma}_i - 1) m_i + N_i (\tilde{\gamma}_i - 1) m_e + \frac{W_B}{c^2} \right] = -\frac{\pi r_o^2}{c^3} \left[ T_{o,2}'/T_{o,2} \right] - \frac{r_o}{c^3} \int (syn + ssc + com), \tag{2.10}$$

where $W_B = (r_o^3/4c^2)R^2 \Delta Z'(B')^2$ is the magnetic field energy in the front. The magnetic field $B'$ is discussed below. Because the acceleration process(es) in the front is not known, we make the well-known supposition (for example, Pacholczyk [15]) that the kinetic energy in the ions is a constant factor $k$ times that in the leptons, $N_i(\tilde{\gamma}_i - 1)m_i = kN_i(\tilde{\gamma}_i - 1)m_e$. As a result, $\tilde{\gamma}_i$ is determined in terms of $\tilde{\gamma}_e$. Therefore, in the following $\gamma$ without a subscript refers always to the lepton Lorentz factor.

A Lorentz transform gives $T_{o,2}' = \Gamma^2[(1 + U^2)T_{o,2} - UT_{o,2} - UT_{o,2}]$, so that

$$-\frac{\pi r_o^2}{c^3} \left[ T_{o,2}' \right] = \frac{r_o^2}{4c^2} B_2^2 \Gamma^2 \left\{ \left[(1+U^2)V_2 - U(1+V_2^2)\right] H_2 - b^2 \left[(1+U^2)V_1 - U(1+V_1^2)\right] H_1 \right\}. \tag{2.11}$$
Dividing equation (2.10) by $\Delta M_o$ gives
\[ \frac{\Gamma}{\Delta M_o} \frac{d}{dT} \left[ N_i (\gamma_i - 1)(1 + k)m_e + \frac{W_B'}{c^2} \right] = 2\mu \Gamma^2 \int \left\{ \left(\ldots\right) \right\} - \frac{r_o}{\Delta M_o c^3} \int (\text{syn} + \ldots), \tag{2.12} \]
where the brackets $\{\}$ denote the same quantity as in equation (2.11).

The synchrotron energy loss rate in the front frame is:
\[ \int \text{syn} = \frac{32\pi c}{9} \gamma_i^2 N_i \left( \frac{B'}{B} \right)^2, \]
where $r_o \equiv e^2/(m_ec^2)$ is the classical electron radius, and $(\gamma_i)^2 = \int_{\gamma_i}^{\infty} d\gamma \gamma^2 f_1/N_i$, where $\gamma_i$ is the Lorentz factor below which synchrotron self-absorption becomes strong as discussed below. The synchrotron self Compton energy loss rate is:
\[ \int \text{ssc} = \frac{4}{3} (N_i \sigma_T \gamma_i)^2 c \left( \frac{54 N_i \gamma_i}{9 \pi^2 \beta \Delta Z} \right) \left( \frac{\beta'}{\beta} \right)^2. \]

The energy loss rate in the front frame due to Compton scattering off of the background photons is:
\[ \int \text{com} = \frac{4}{3} (N_i \sigma_T \gamma_i)^2 c \int \text{ssc} \left( \frac{U'_\text{ph}}{U_{\text{ph}}} \right), \]
where $U'_\text{ph}$ is the background photon energy density in the front frame.

The lepton distribution suggested by observations has a hard power law form in the main energy containing range, say, $f_1 = K_1/\gamma^2$ for $\gamma_1 \leq \gamma \leq \gamma_2$ with $1 \ll \gamma_1 \ll \gamma_2$. For $\gamma \geq \gamma_2$, the distribution is steeper, say, $f_1 = K_2/\gamma^3$, and at even larger energies, $\gamma \geq \gamma_3$, $f_1$ is even steeper. For $\gamma < \gamma_1$, $f_1$ is assumed negligible. Thus $f_1$ is characterized mainly by $\gamma_1$ and $\gamma_2$. We have $\bar{\gamma} = \gamma_1[1 + \ln(\gamma_3/\gamma_1)]$, and $\bar{\gamma}^2 = C\gamma_1\gamma_2$, with $C = 1 + \ln(\gamma_3/\gamma_2)$. The relevant value of $\gamma_1$ is the Lorentz factor below which synchrotron self-absorption becomes strong. Self-absorption as treated for example by Pacholczyk [15] then gives $\gamma_1 = \gamma_1(B', n_i', R)$. With $\gamma_1$ known, we have $\gamma_2 = \gamma_1 \exp(\bar{\gamma}/\gamma_1 - 1)$. The value of $C$ depends only weakly on $\gamma_3/\gamma_2$ and we take $\gamma_3/\gamma_2 = \bar{\gamma}/\gamma_1$.

**Magnetic flux conservation:** The magnetic field in front frame $B'$ is determined by taking into account the influx of magnetic flux with the new matter (subscript 2) and the old matter (subscript 1). We let $\Phi' = R \Delta Z' B'$ denote the toroidal flux (in Gauss) in the front. Then
\[ \frac{d\Phi'}{dT} = B_o \left[ \Delta V_2 H(\Delta V_2) + b \Delta V_1 H(\Delta V_1) \right], \tag{2.13} \]
where $B_o \equiv (B_2)_{z=0}$ and the $H$'s are the same as in equation (2.2).

**3. Results**

Equations, (2.2), (2.3), (2.8), (2.12) and (2.13) have been solved numerically to obtain the time dependences of all of the physical variables, for example, $N_i(N_i, \Gamma, \bar{\gamma}_i, B')$. The apparent synchrotron, SSC, and inverse Compton luminosities for an observer at an angle $\theta_{\text{obs}}$ to the line of sight are calculated including the solid angle boost factor $\delta^2$ between the front frame and the observer, where $\delta = \Gamma^{-1}[1 - U\cos(\theta_{\text{obs}})]^{-1}$. The observed frequency is boosted by a factor $\delta$. Figure 2 shows sample results.

At early times in an outburst, the SSC radiation predominates because the magnetic field in the front is strong and the number of synchrotron photons is higher than that of background photons. Also, the pair production rapidly gives $f_{\text{pp}} \gg 1$ in the front. Later, the magnetic field in the front decreases and thus the background photons predominate.

At the same time, the leptons are accelerated in the front and consequently inverse Compton radiation grows, and then later decreases. The time of the peak of the inverse Compton emission depends approximately linearly on the radius $r_{\text{ph}}$ of the background photon distribution. Synchrotron photons are radiated at much lower luminosity during all times of the front propagation. The radio flux becomes visible as soon as the observer's frequency of self-absorption (corresponding to $\gamma_1$) decreases into the radio band. The radio emission also may be attenuated due to free-free absorption by distributed hot gas (e.g., Matveenko et al. [16]).
In the case shown in figure 2, the apparent velocity of the front saturates rapidly (after observer’s time $t'' = 0.02d$) at $v_{app} = 8.9c$, the bulk Lorentz factor at $\Gamma = 14.6$, and $\delta = 3.1$; and thus the lab time is $t = 45t''$. The parameters for this case are $M = 3 \times 10^8 M_\odot$ so that $r_o = 2.67 \times 10^{14} cm$, $\mu = 330$, $B_o = 700 G$, $b = 0.12$, $\Gamma_1 = 5$, $\Gamma_2 = 25$, $\nu = 0.1$, and $k = 1$. For the background photons, $L_{UV} = 10^{46} erg/s$, $\bar{\epsilon}_{ph} = 10 eV$, and $r_{ph} = 0.1 pc$. Also, we have taken $f_{hi1} = f_{hi2} = 1$. Pair production in the front causes $f_{hi}$ to increase rapidly and gives $f_{hi} \approx const. = 740$ for $t'' > 0.03d$. Faraday rotation within the front is negligible at all times at centimeter wavelengths. The ratio of the particle to field energy in the front rapidly saturates at about 21 so that magnetic pinching is negligible. The maximum of the SSC emission occurs at $t'' = 0.028d$, that of the synchrotron at $0.068d$, and that of the Compton at $1.8d$. 

At time $a$ in the figure, the lower limit of the observed synchrotron spectrum due to self-absorption (corresponding to $\gamma_1$) is $10^{12} Hz$ while the upper limit (corresponding to $\gamma_3$) is $10^{16} Hz$. At time $b$ these frequencies are $10^{11}$ and $10^{16}$. For longer times, the self-absorption frequency decreases approximately as $(t'')^{-0.9}$. At the peak of the Compton emission, the observed frequencies are between $10^{21}$ and $10^{26}$. At $t'' = 10d$, the SSC emission extends from about $10^{15}$ to $10^{25}$.

Both authors were supported in part by NSF grant AST-9320068. MMR was also supported in part by the Scientific and Educational Center of Kosmomicrophysics "KOSMION" and by Russian Fundamental Research Foundation Grant No 93-02-17106. RVEL thanks D.E. Harris for stimulating discussions.

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