The Backreaction Problem

Eanna Flanagan, Cornell
SUMMARY OF TALK

WHERE DO WE STAND ON BEING ABLE TO COMPUTE THE BACK REACTION OF EMITTED GRAVITATIONAL WAVES ON THEIR SOURCES?

- BLACK HOLE MERGERS
- STELLAR PULSATIONS
- SETTLING DOWN OF BLACK HOLES

SOURCES FOR WHICH BACK-REACTION IMPORTANT

- BINARY WITH MM & $l \leq 1$
- BINARY WITH VNC & MCM
  - LISA
  - LIGO

RECENT PROGRESS

OPEN PROBLEMS

QUESTIONS OF PRINCIPLE

DEVELOPING COMPUTATIONAL METHODS

APPLYING COMPUTATIONAL METHODS

ODES

PDES
**COMPARABLE MASS BINARIES**

NS/NS, NS/BH & BH/BH BINARIES with masses ~1MO - 20MO

Highly accurate templates needed for inspiral waveforms, especially for higher mass systems.

**POST-NEWTONIAN EXPANSIONS**

Methods of improving convergence, motivated by examination of $M/M<1$ limit

Pade approximants effective one-body potential

(Damour, Iyer, Sathyaprakash)

**STATUS OF POST-NEWTONIAN EXPANSIONS**

Post-2-Newtonian, point particle templates have been the standard for several years

(Blanchet, Damour, Iyer, Will, Wiseman, 1995)
STATUS OF POST-NEWTONIAN EXPANSIONS

POST-3-NEWTONIAN EQUATIONS OF MOTION OBTAINED BY DAMOUR, JARANOWSKI & SCHAER. USED ADM HAMILTONIAN WITH 8 FUNCTION SOURCES. REGULARIZATION METHOD GAVE TWO, UNDETERMINED NUMERIC PARAMETERS WHICH WERE SUBSEQUENTLY DETERMINED BY (i) DEMANDING GLOBAL Poincaré INVARIANCE, AND (ii) USING DIMENSIONAL REGULARIZATION.

JS, PRD 57 7274 (1998)
DJS, gr-qc/9912092
gr-qc/0003051
gr-qc/01005038

BLANCHET ET AL. HAVE COMPUTED POST-3+5-NEWTONIAN WAVEFORMS UP TO ONE UNDETERMINED PARAMETER (AGAIN A REGULARIZATION AMBIGUITY).

L. Blanchet, G. Faye, B. Iyer, B. Sjuts, gr-qc/0105099

ONGOING RESEARCH BY C. WILL & PATI, STARTING FROM FINITE FLUID BALLS, SHOULD RESOLVE REGULARIZATION AMBIGUITIES AT POST-3 ORDER.

POINT PARTICLE EQUATIONS OF MOTION WELL DEFINED UP TO POST-4.5-ORDER (DAMOUR, 1987).
ANALYSIS OF WAVEFORMS PRODUCED IN THE TRANSITION FROM INSPIRAL TO PLUNGE SHOULD BOOST SIGNAL-TO-NOISE RATIO BY ~50%.

A. Buonanno and T. Damour, gr-qc/0011052
A. Ori and K.S. Thorne, gr-qc/0003032

KEY OPEN PROBLEM: EXTEND TREATMENT OF SPIN EFFECTS TO HIGHER ORDER.

Tagoshi, Ohashi & Owen (2000) have computed translational equations of motion to post-2.5-order. Equations for evolutions of spins known only to 1.5-order.

D. Chernoff numerically exploring solutions and waveforms.
**Comparable Mass Binaries & The Self-Force on Point Particles**

\[ M \sim 10^6-10^8 M_\odot \]

__Inspiral of compact objects (NS/BH) into supermassive black holes a key source for LISA.__

See L.S. Finn and K.S. Thorne, 85-90/000 7074 for signal-to-noise estimates and eventrates.

__Inspiral of compact objects into intermediate mass black holes (\( \sim 200 M_\odot \)) a possible source for LIGO (more speculative).__
HIGH ACCURACY THEORETICAL MODELING NEEDED:

- Required $\Delta \varpi / \varpi \sim \frac{1}{N_{\text{cycles}}}$

- $E_{\text{lost}}, 1\text{ orbit} \propto M$
  
  $E_{\text{orbit}} \propto M$
  
  $N_{\text{cycles}} \propto \frac{1}{M} \propto \frac{M_{S}}{M}$

- Some example numbers, for rapidly spinning black holes

<table>
<thead>
<tr>
<th></th>
<th>$M$</th>
<th>$M_{S}$</th>
<th>$8M_{S}$</th>
<th>$M_{S}$</th>
<th>$\sim 600$</th>
<th>$\sim 50\text{ yrs}$</th>
<th>$\frac{1}{M_{S}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIGO</td>
<td>$1M$</td>
<td>$3M$</td>
<td>$8M$</td>
<td>$M$</td>
<td>$M_{S}$</td>
<td>$\frac{1}{M_{S}}$</td>
<td>$\frac{1}{M_{S}}$</td>
</tr>
<tr>
<td>LISA</td>
<td>$1M$</td>
<td>$10^{6}M$</td>
<td>$3.5M$</td>
<td>$M_{S}$</td>
<td>$M_{S}$</td>
<td>$\frac{1}{M_{S}}$</td>
<td>$\frac{1}{M_{S}}$</td>
</tr>
</tbody>
</table>

(Finn & Thorne, 2000)

- So require $\Delta \varpi / \varpi \leq 10^{-6}$ for LISA

Applications of Measurement

- Measure multipole moments of space-time geometries
  
  & test no-hair theorem.

- Test for scalar component of gravity
THEORETICAL CHALLENGE: COMPUTE ORBITAL EVOLUTION AND WAVEFORMS TO FRACTIONAL ACCURACY OF $10^{-6}$, FOR GENERIC ORBITS (SPINNING BLACK HOLE, INCLINED, ECCENTRIC ORBIT).

OVER THE LAST FEW YEARS, ~50 PAPERS, 4 DEDICATED CONFERENCES ("CARRA RANCH").

PROBLEM NOT YET SOLVED, BUT LOTS OF PROGRESS.

BASIC METHOD: EXPANSION IN $\mu/m$ ($\mu/m$ OF ORDER UNITY) USING BLACK HOLE PERTURBATION THEORY (TENKOZSKY - SASAKI - NAKAMURA FORMALISM).

TWO VARIANTS

- CONSERVATION LAW METHOD
- LOCAL FORCE METHOD
Method for Calculating Orbital Evolution

- Use adiabatic approximation:
  \[
  \frac{\text{inspiral}}{\text{torque}} = \left( \frac{v}{c} \right)^5 \left( \frac{M_{\text{BH}}}{M_{\odot}} \right) \sim 1
  \]

- Geodesic orbits characterized by 3 conserved quantities:
  - Energy \( E \)
  - Angular momentum \( L_z \)
  - Carter constant \( C \)

- Idea: Calculate \( \frac{dE}{dt} \), \( \frac{dL_z}{dt} \), \( \frac{dc}{dt} \) and infer slow evolution of orbit

- Details:
  \( G_{\text{BH}} = G_{\text{Kerr}} + L_{\text{Kerr}} (t, r, \theta, \phi) \)

- Einstein equation \( \rightarrow \) PDE for \( L_{\text{Kerr}} \)
  - Multi polar decomposition \( \rightarrow \) ODES (Teukolsky eqn)

- Use geodesic orbit as source, calculate
  \[ \left( \frac{dE}{dt} \right)_\phi \text{ waves}, \left( \frac{dL_z}{dt} \right)_\phi \text{ waves} \]

- Failure of method:
  Cannot calculate \( \frac{dc}{dt} \)
  Method works for equatorial orbits (\( c=0 \)) or for circular orbits.
  For generic orbit, method fails...
CONSERVATION LAW METHOD (CONT.)

OTHER DRAWBACKS OF METHOD

1) WORKS ONLY WHEN SPACETIME HAS SYMMETRIES
2) CONSERVATIVE PIECE OF SELF FORCING OMITTED.

VERY USEFUL FOR EXPLORING WAVEFORM BEHAVIORS HAS BEEN WIDELY APPLIED.

\[ q = 0 \]

Tanaka, Shibata, Sasaki, Togoshi, Nakamura (1993)

Mino, Tanaka, Shibata, Sasakai, Togoshi, Nakamura (1998)

C. Cutler, E. Poisson, D. Kennehick (1994)

\[ q \neq 0 \]

Finn and Thorne (2000)

D. Kennehick (1998)


CONVENTIONAL WISDOM: CANNOT BE EXTENDED TO GENERIC ORBITS.

But

- Hughes suggests evolving using the constraint \( i = \text{fixed} \).
- There might exist additional conserved quantities. For a scalar field on Kerr \( \langle \mathcal{Q} \rangle = \sum_{m=0}^{\infty} \sum_{l=0}^{\infty} \sum_{2l+1}^{\infty} e^{il(l+1) \phi} \omega^{m} \mathcal{Q}^{n} \rangle \) is conserved and non-vanishing for odd \( n \).

(A. Ashtekar, N. Tichy, ...)
Local Force Method

Basic idea is to obtain a local force expression and corresponding equation of motion that is analogous to Abraham–Lorentz–Dirac equation for charged particles:

\[ q = \frac{1}{2} \dot{v} + \frac{1}{3} \frac{q^2}{mc^2} \]

Complications in curved space time

Back scattering

Now - dissipative contributions to self force

Interested in gravitational self force, but scalar & electromagnetic self forces useful as toy models.
**Gravitational Case:**

- Isolated body moving in vacuum gravitational field
  - \( L_{\text{ext}} = \frac{1}{c^2} \nabla_i \nabla^i \Phi \)
  - \( M \ll L_{\text{ext}} \) and \( L \ll L_{\text{ext}} \)
  - \( G = c = 1 \)

\[
F^\alpha = M^\alpha = M_\alpha + O\left( \frac{M^2}{L^2} \right) + O\left( \frac{L^p}{L_{\text{ext}}} \right)
\]

- Geodesic motion
- Independent of internal structure
  - Self Force

\[
= M^2 \int_{\text{self}} [ \Phi, Y_{\text{self}}, J_{\text{ab}} ]
\]
RETARDED FIELD OF POINT SOURCE

\[ x^i = r \eta^i \]

- Use Riemann Normal coordinates at \( P \)
- \( \mathbf{u}, \mathbf{\dot{u}}, \mathbf{\ddot{u}} \) evaluated at \( P \)

\[
F_{\alpha\beta}(x) = \frac{1}{r^2} \left\{ 2 \mathbf{\hat{a}} \cdot \mathbf{u}_{[\alpha} \mathbf{h}_{\beta]} \right\}^2 \\
+ \frac{1}{r} \left\{ -\mathbf{\dot{a}} \cdot \mathbf{u}_{[\alpha} \mathbf{u}_{\beta]} + \frac{1}{2} \frac{\partial}{\partial x^c} \mathbf{u}_{[\alpha} \mathbf{u}_{\beta]} \right\} \\
+ \frac{1}{r^2} \left\{ 2 \mathbf{\hat{a}} \cdot \mathbf{\dot{u}}_{[\alpha} \mathbf{u}_{\beta]} \right\} \\
+ \mathbf{u}_{[\alpha} \mathbf{u}_{\beta]} R_{\gamma \delta} \mathbf{u}^{\gamma} \mathbf{u}^{\delta} + \text{terms with all numbers of } \mathbf{\hat{a}}'s \}
\]

\[ + O(\epsilon) + \int_{\mathbb{R}^2} \mathbf{u}^{\alpha} \mathbf{e}^{\mu\nu} \left( \mathbf{u}^{\nu} \frac{\partial}{\partial x^\mu} - \frac{1}{2} \frac{\partial}{\partial x^\nu} \right) \mathbf{u}^{\alpha} \mathbf{e}^{\lambda\eta} \left( \mathbf{u}^{\eta} \frac{\partial}{\partial x^\lambda} - \frac{1}{2} \frac{\partial}{\partial x^\lambda} \right) \]
METHODS FOR DERIVING SELF-FORCE FORMULA

EM = ELECTROMAGNETIC CASE
GW = GRAVITATIONAL CASE

- Local energy-momentum balance, world-tube construction.
  - Dirac (1920's)
  - deWitt and Brehme (1960)
  - Hobbs (1968)
  - Mino, Sasakai, Tanaka (1997)

- Matched asymptotic expansion
  - Mino, Sasakai, Tanaka (1997)
  - Detweiler (2000)

- "Axiomatic" approach
  - Quinn and Wald (1997)

- Symmetry & Dimensional arguments
  - Ori (unpublished)

All these methods give the same results.
**Electromagnetic Self-Force**

Context:
- Arbitrary worldline, $x^a = z^a(t)$
- Arbitrary metric, $\text{arbitrary field } F^{a\beta}$
- Incoming Maxwell field $F_{\text{in}}$

\[ F^a_{(\text{in})} = Q F_{\text{in}}^\alpha u_\alpha + \frac{2}{3} Q^2 (g^{ab} + u^a u^b)(\nabla_\mu \tilde{a}_\rho)(\nabla_\rho \tilde{a}_\mu) \]

"$\tilde{a}$" term. Reduces to using "$\frac{1}{2} (\text{Advanced- Retarded})$" in flat space time.

\[ + \frac{1}{3} Q^2 (g^{ab} + u^a u^b) R_{\beta\gamma} u^\beta u^\gamma \]

\[ + Q^2 u_\beta \lim_{\epsilon \to 0} \int_{t_0 - \epsilon}^{t_0 + \epsilon} dt' \sqrt{g} G^a_{\beta\gamma \alpha\mu} \text{ ret} \left[ z(t), z(t') \right] u_\gamma, (t') \]

$Q^2 F^{a\beta} u_\beta$

Freely falling charge:

Newtonian analysis: \[ q = -\nabla \cdot q, \quad \vec{F} = \frac{q}{m} \vec{v} \]

Relativistic analysis: \[ q^a = 0 \]

but tail term reproduces Newtonian case.

(de Witt & de Witt, 67).
GRAVITATIONAL SELF-FORCE

Context: Freely falling particle in gravitational field, no other matter.

Compute total force of radiation emitted in red region

"Turn off" mass of particle here

Force at $\vec{r} = \lim_{\Delta t \to 0} F^a(\Delta t)$

Find:

$F^a(\Delta t) = \frac{1}{2} m^2 \left( g^a b + u^a u^b \right) \left[ \nabla_\alpha h^\alpha_{\sigma} - \nabla_\sigma h^\alpha_{\alpha} - \nabla^\sigma h_{\alpha\beta} \right]$

where

$h^a_{\sigma}(x) = \lim_{\Delta t \to 0} \int_{-\infty}^{\infty} d\tau' \left( g_{\mu\nu} \phi^\mu (x') \phi^\nu (x') \right)$

Equivalent formulation

$h^a_{\sigma} = h^a_{\sigma}^{\text{ret}} - h^a_{\sigma}^{\text{tail}}$

(Detweiler, 2000)
(Brady, Anderson et al.)

Specific identification of as power series expansion
ISSUES OF PRINCIPLE: RECENT DEVELOPMENTS

**Generalization to Non-Vacuum Space-times Given by Poisson (2001), Necessary in Order to Give the Correct Result in the Post-1-Newtonian Limit.**

**Generalization from Retarded Harmonic Gauge to Arbitrary Gauge Given by Barak and Ori (2001).**

ISSUES OF PRINCIPLE: FUTURE TASKS

**Generalize the Matched Asymptotic Expansion Analysis of Mino et al. to an Arbitrary Extended Object Rather than Black Hole.**

**Derive the Quinn-Wald Axioms from the Matched Asymptotic Approach in the Scalar, Electromagnetic & Gravitational Cases.**
Matter field $h$

\[ h_h \]

\[
\begin{align*}
\text{Matter perturbation } & S_4 \text{ and } S_{\text{gap}} = -\hbar \beta \\
\text{coupled at linear order.} \\
\text{Combined Green's function} \\
\text{"Matter mediated force"}
\end{align*}
\]

\[
G = \begin{bmatrix}
G_{hh} & G_{h4} \\
G_{4h} & G_{44}
\end{bmatrix}
\]
Efforts to Develop a Practical Computational Scheme to Implement the MSTQW Prescription

It is difficult to compute the tail field using the standard Teukolsky mode decomposition formalism. Attention has focused on finding ways to subtract the singular part.

Mode Sum Regularization

\[ F_n = \sum \left\{ F_{n,\text{bare}} - A_n^p - B_n - \frac{C_n}{n^2} \right\} \]

A. Ori, L. Burko, L. Buchbinder, Y. Mino, H. Nakano, M. Sasaki
Works for GRAV. Force in Schwarzschild.

Power Expansion Regularization

Y. Mino

3-Function Regularization

C. Lousto

Open Problems (Grav. Case Only)

- Transforming self force from harmonic gauge to Regge-Wheeler or radiation gauges is still ill-defined (Ori & Barack, 2001)
To compute the bare force for each multipole, we need to recover $h_{\alpha \beta}$ from $Y_4$.

Formalism of Chrzanowski (1975) needs to be adapted/extended.